

# Numerical study of the periodic Anderson model with a quarter-filled conduction band

Shuxiang Yang

Feb. 15, 2015

Collaborators:

Juana Moreno  
Mark Jarrell



# Outline

- Model
  - Periodic Anderson model
  - Relations with the Kondo lattice model and Hubbard model
- Numerical methods:
  - Dynamical mean-field theory (DMFT)
  - Dynamical cluster approximation (DCA)
  - Dual fermion method (DF)
- Numerical results for 2D and 3D lattice
  - Bench-mark DF with DMFT and DCA results
  - Phase-diagrams
  - Origin of the CDW ordering
- Conclusion

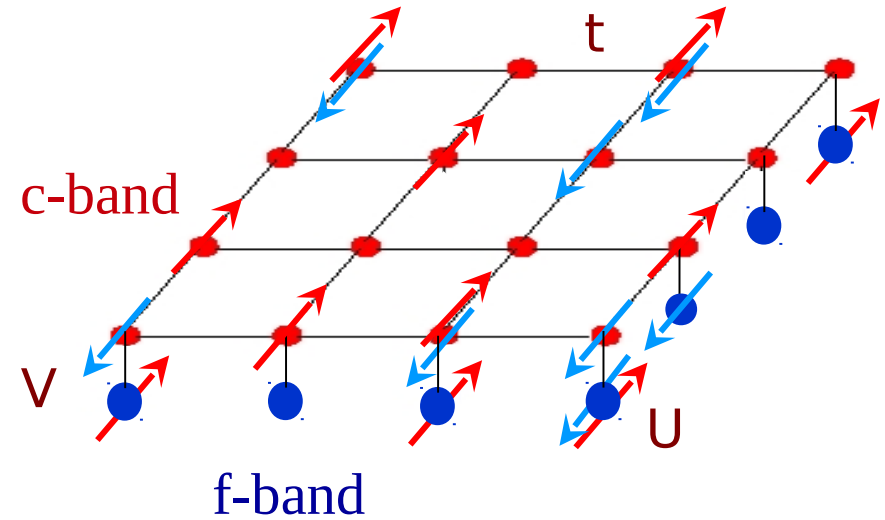
# Model

- Periodic Anderson model:

$$\hat{H} = -t \sum_{\langle ij \rangle, \sigma} (\hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + \hat{c}_{j\sigma}^\dagger \hat{c}_{i\sigma})$$

$$+ V \sum_{i\sigma} (\hat{c}_{i\sigma}^\dagger \hat{f}_{i\sigma} + \hat{f}_{i\sigma}^\dagger \hat{c}_{i\sigma})$$

$$+ \epsilon_f \sum_{i\sigma} \hat{f}_{i\sigma}^\dagger \hat{f}_{i\sigma} + U \sum_i \hat{n}_{f,i\uparrow} \hat{n}_{f,i\downarrow}$$



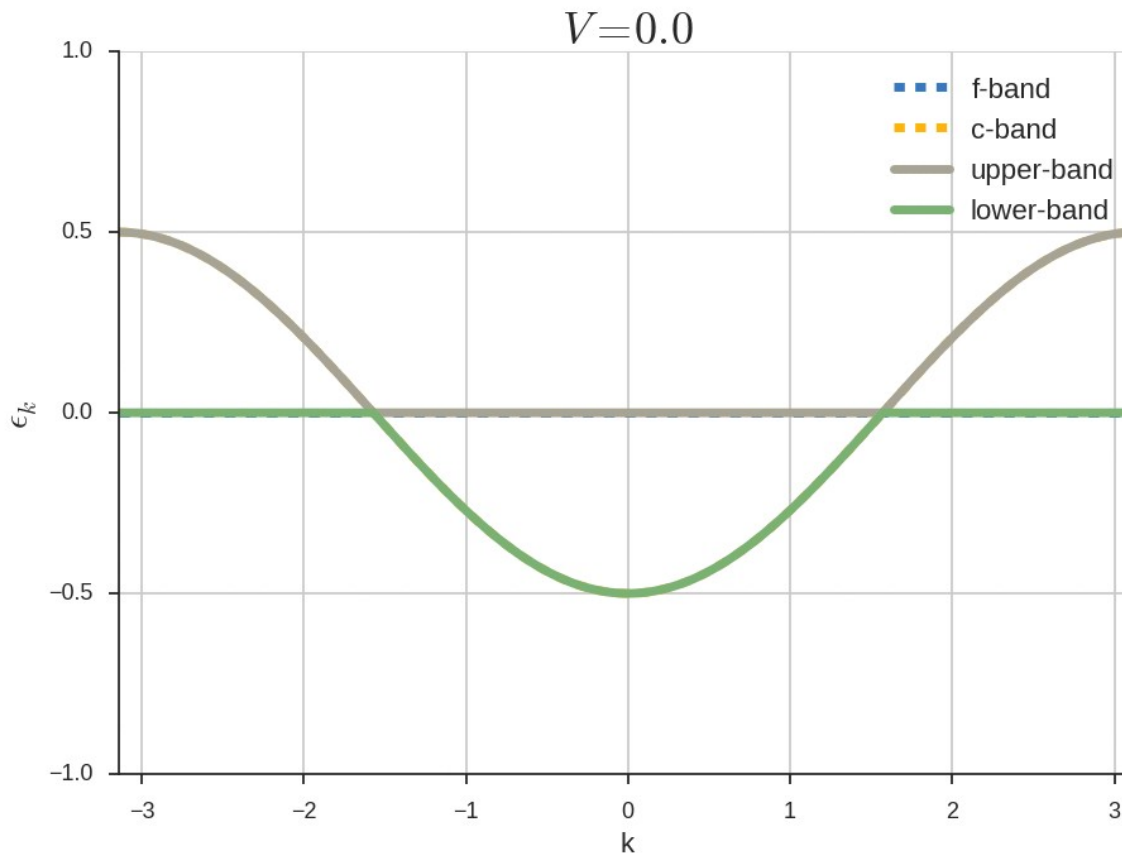
- Two-band model
- Conduction band: c-electron can hop, no interaction
- Localized band: f-electron cannot hop, Coulomb interaction
- Hybrid through  $V$  term

# Model

- $U=0$

$$\epsilon_{upper-band}(k) = \frac{\epsilon_k + \sqrt{\epsilon_k^2 + 4V^2}}{2}$$

$$\epsilon_{lower-band}(k) = \frac{\epsilon_k - \sqrt{\epsilon_k^2 + 4V^2}}{2}$$



# Related to other models

- Kondo lattice model (freezing out f-electron charge fluctuations):

$$U \gg t, V \quad n_{f\uparrow} + n_{f\downarrow} = 1$$

$$\hat{H} = -t \sum_{\langle ij \rangle, \sigma} (\hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + h.c.) + J \sum_i \hat{\mathbf{s}}_i \cdot \hat{\mathbf{S}}_i$$

- Hubbard model (integrating out c-electron formally):

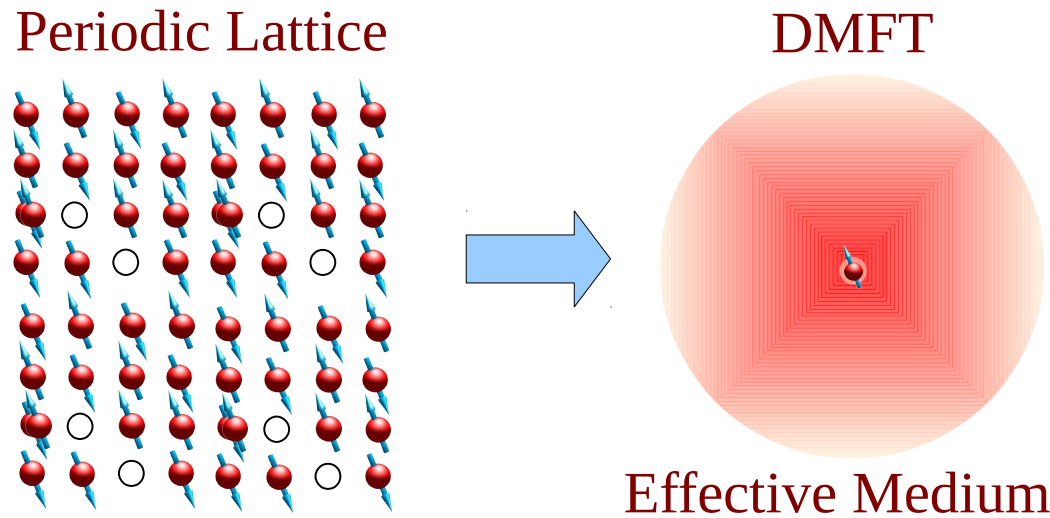
$$S[f^*, f] = - \sum_{k\omega\sigma} f_{\omega k\sigma}^* G_0(k, i\omega)^{-1} f_{\omega k\sigma} + U \sum_i \int_0^\beta d\tau n_{f,i\uparrow}(\tau) n_{f,i\downarrow}(\tau)$$

$$G_0(k, i\omega)^{-1} = i\omega + \mu - \epsilon_f - \sum_k \frac{V^2}{i\omega + \mu - \epsilon_k}$$

Adapting action-based codes from Hubbard model to PAM is easy!

# Numerical methods -- I

- Dynamical mean-field theory (DMFT)



A. Georges, et.al., RMP 68, 13 (1996)

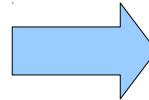
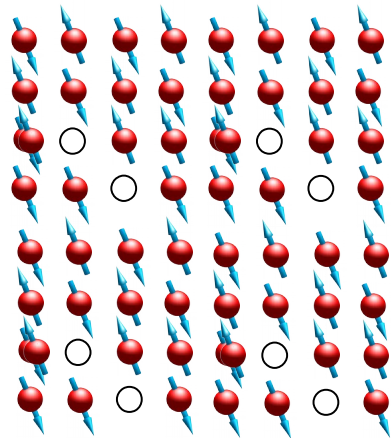
- **Exact** in infinite dimension, **approximate** for finite dimensions
- Local approximation
- A great success. Best for the local physics, e.g. Mott physics
- Missing non-local physics, e.g. pseudo-gap, d-wave superconductivity

Many attempts to improve DMFT

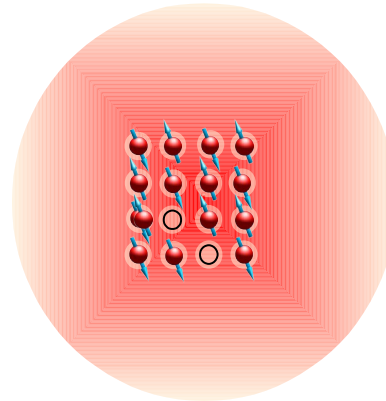
# Numerical methods -- II

- Dynamical cluster approximation (DCA)

Periodic Lattice



DCA



Effective Medium

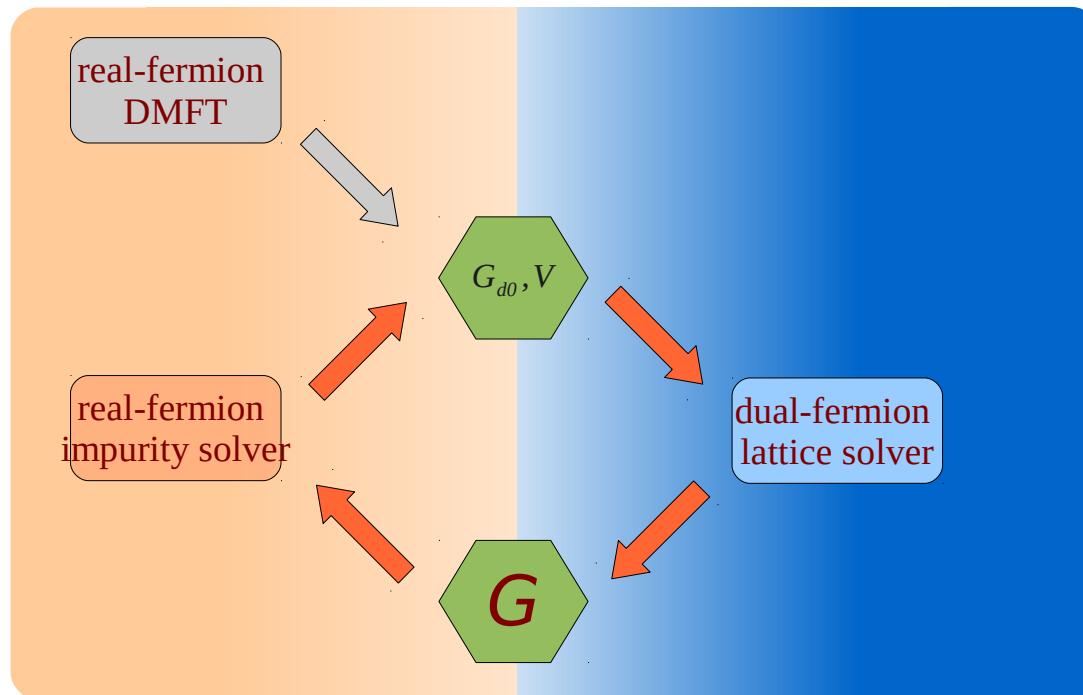
Th. Maier, et.al., RMP. 77, 1027 (2005)

- One of the several proposals to go beyond single-site DMFT description  
similar proposal: cellular-DMFT
- Short-ranged correlations included
- Capture non-local physics, e.g. pseudo-gap, d-wave superconductivity

# Numerical methods -- III

- Dual fermion method

A. N. Rubtsov et al., PRB 77, 33101 (2008)



DF mapping:

$$G_{d0} = G_{lat} - G_{imp}$$

$$V \approx F$$

CT-QMC

**Real fermion**  $\longleftrightarrow$   
(strongly-correlated)

**Dual fermion**  
(weakly-correlated)

Second-order perturbation

- One of the several proposals to expand on DMFT solution
- Similar to small cluster DCA results



# Numerical methods -- IV

- DMFT

- Fast and no minus-sign problem for PAM
- Quick scan of the parameter space

- DCA

- Expensive and has the minus problem
- Check if short-ranged correlations change physics extracted from DMFT
- Bench-mark DF results

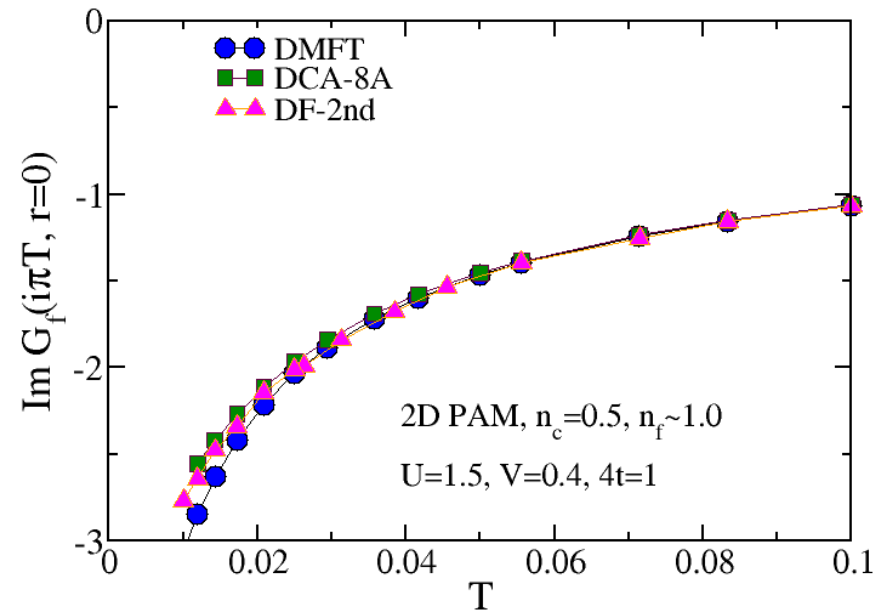
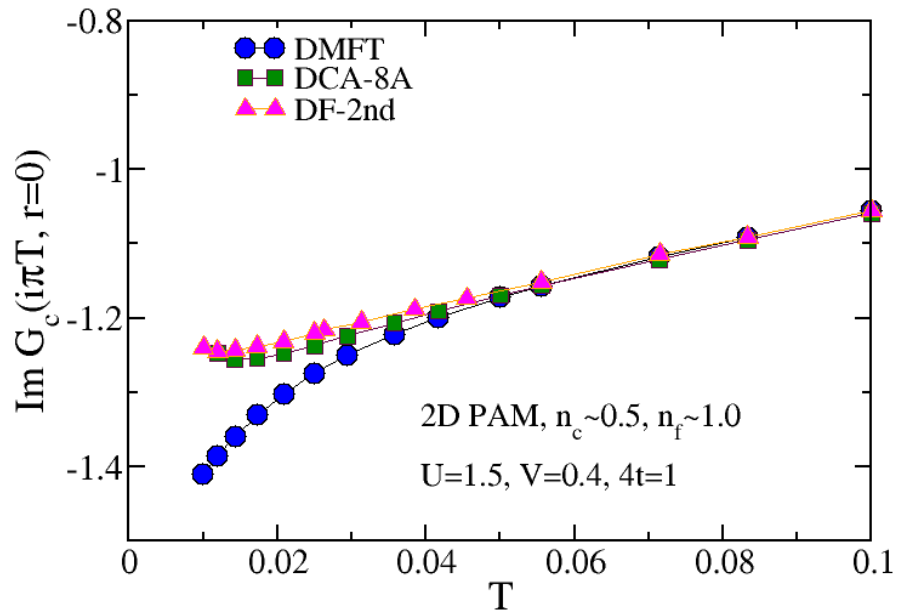
- DF

- No minus-sign, more expensive than DMFT, but less expensive than DCA
- Used when sign problem in DCA is bad and thus DCA is too expensive

# Results

- Bench-mark
  - Compare DF results against DMFT and DF results
- Phase-diagram for 2D and 3D lattice
  - Assume no symmetry broken in the system
  - Divergence of the susceptibility signals phase-transition
- Origin of the CDW ( $q=(\pi,\pi)$ ) ordering

# Comparison of the DCA and DF results

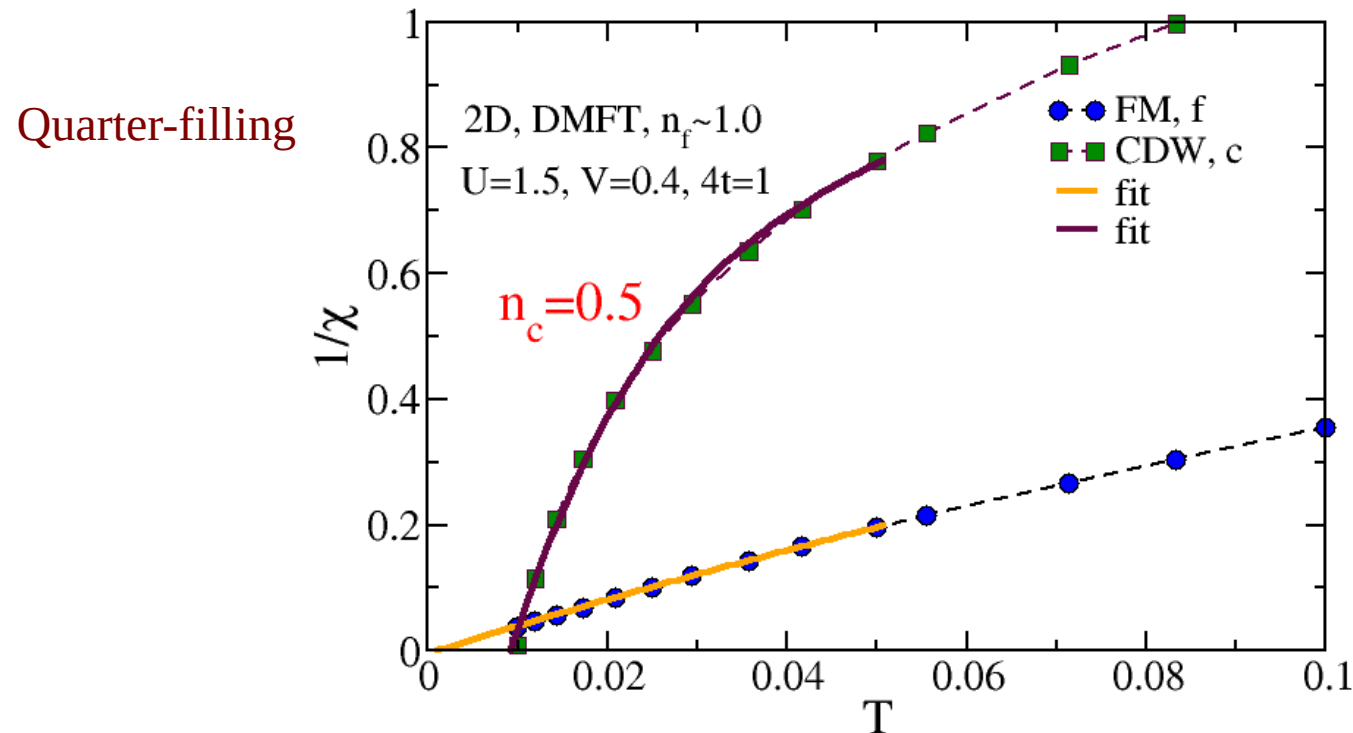


- For both c-band and f-band, DF can capture some essential short ranged correlations as the DCA calculation.

# 2D case

Assume no symmetry broken in the system

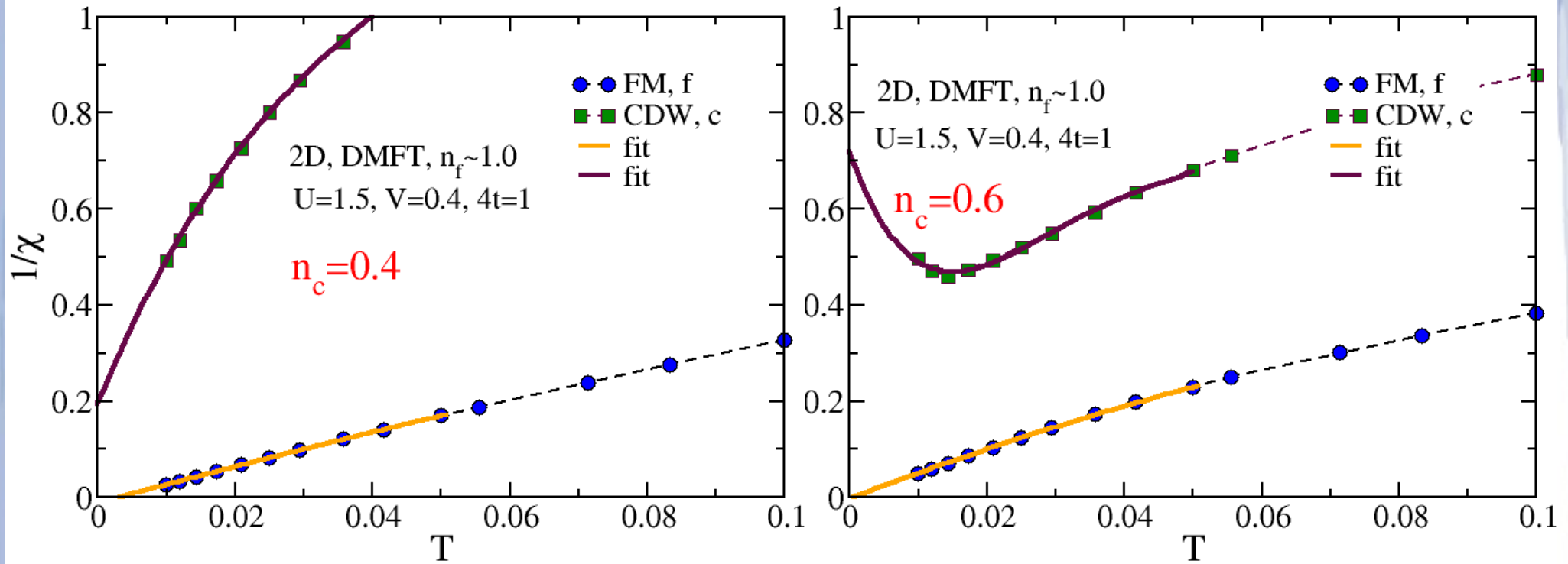
Divergence of the susceptibility signals phase-transition



- Two phase-transitions, ferro-magnetic (FM) and charge-density-wave (CDW,  $q=(\pi,\pi)$ )
- CDW  $T_c$  is higher than FM  $T_c$

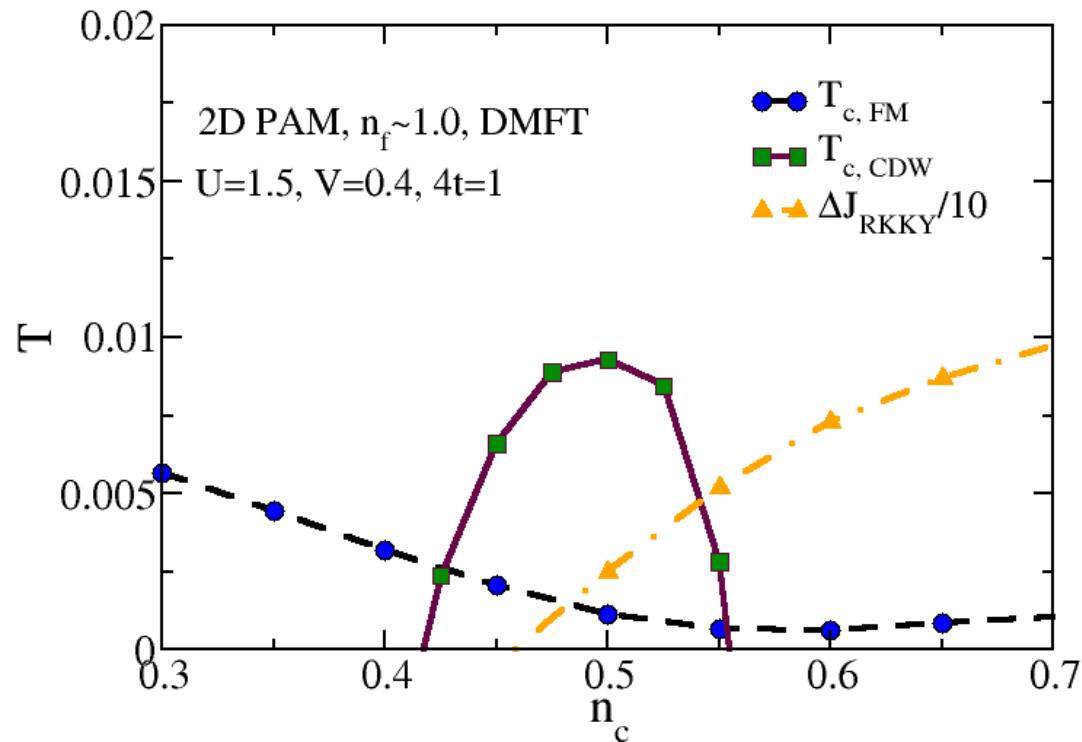
# 2D case

## Off-quarter-filling



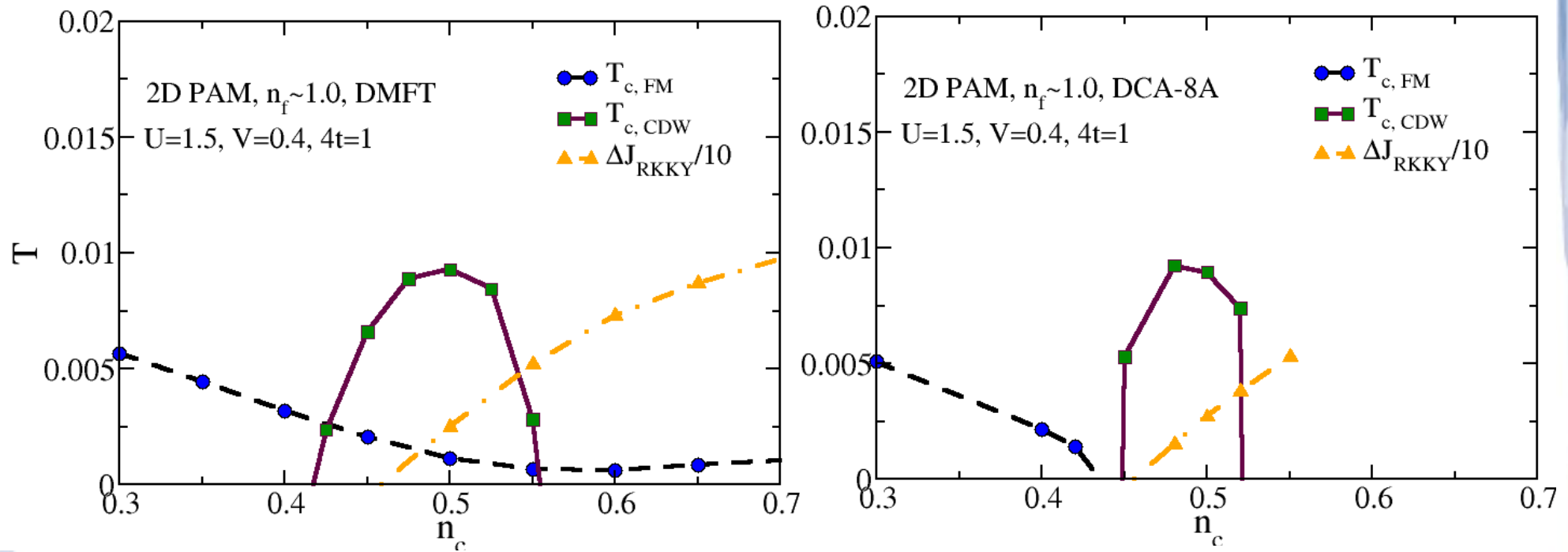
- Going either below or above quarter-filling, only FM diverges

# 2D phase-diagram



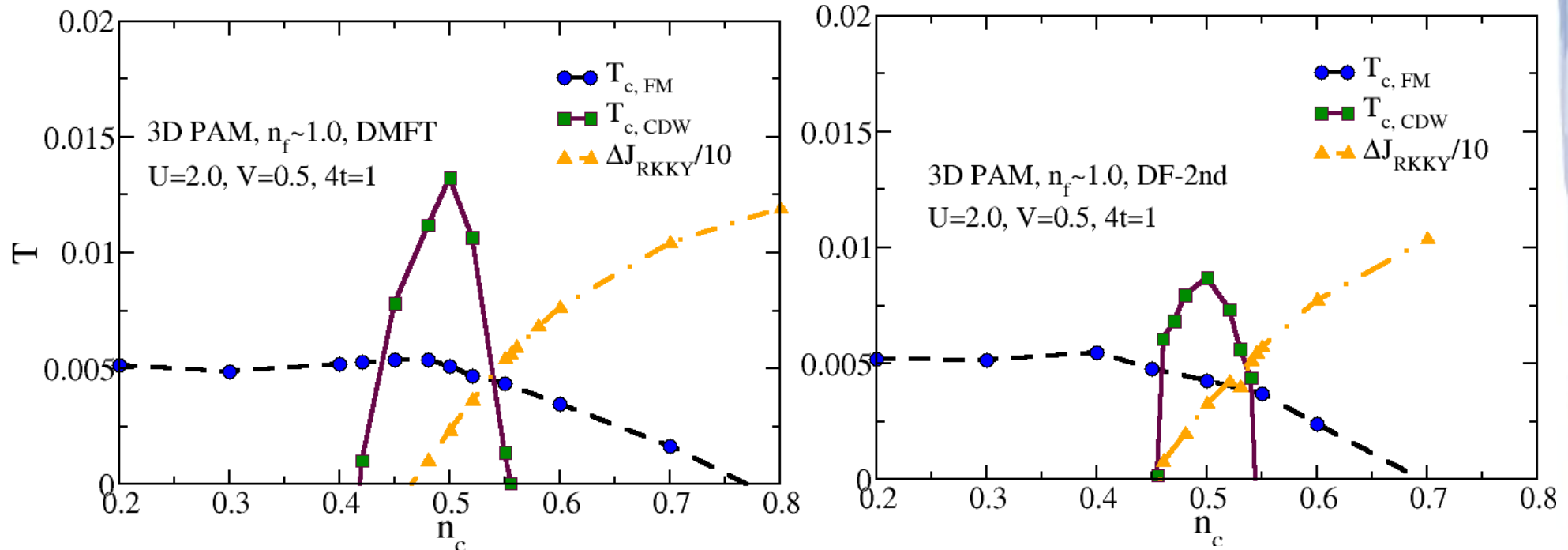
- FM  $T_c$  decreases when increasing c-electron filling
- CDW  $T_c$  has a dome shape around quarter-filling

# 2D phase-diagram



- FM ordering is reduced when short-range correlations are included
- CDW dome persists for DCA-8A results, but with reduced size

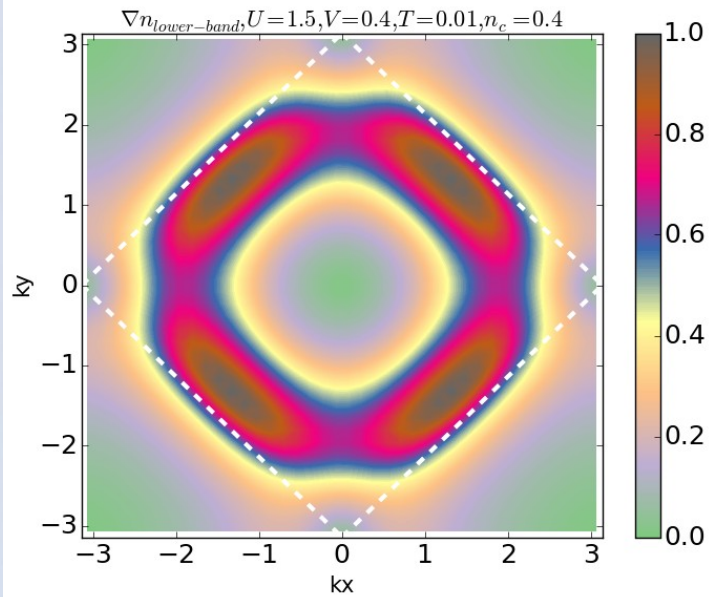
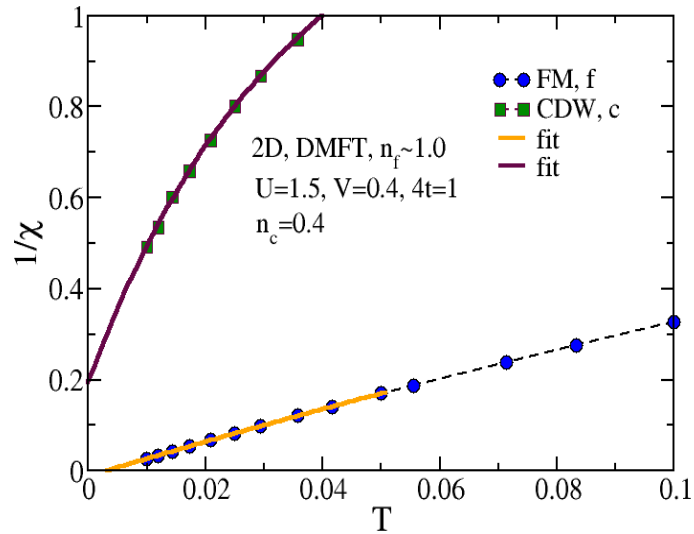
# 3D phase-diagram



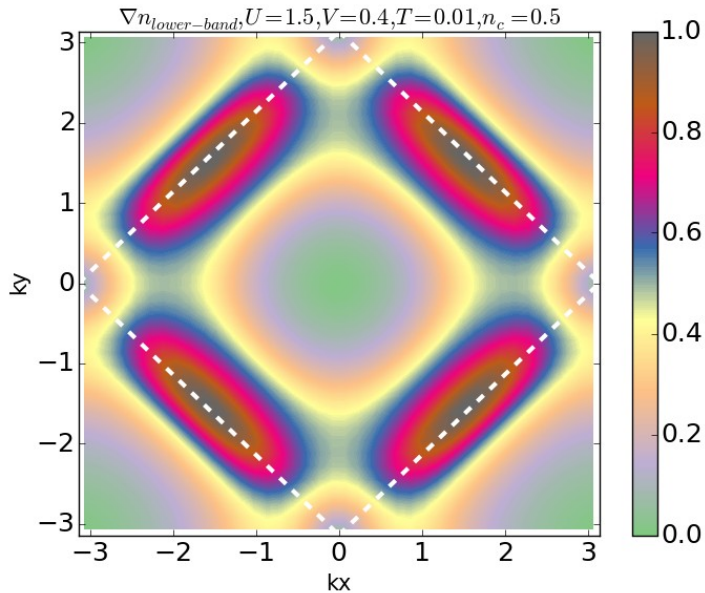
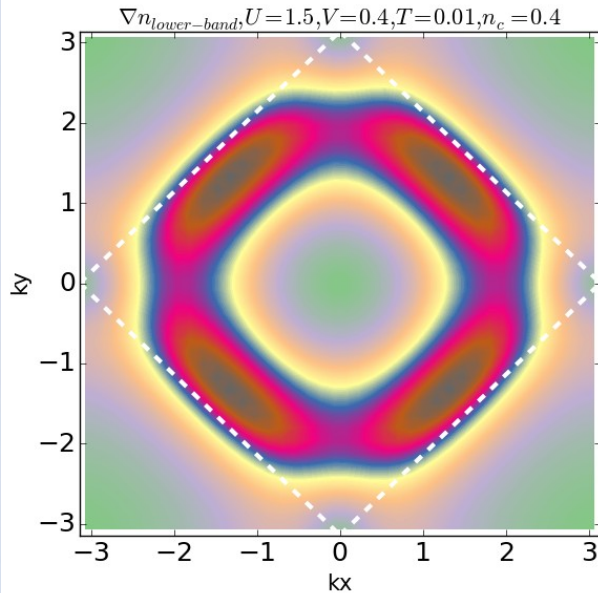
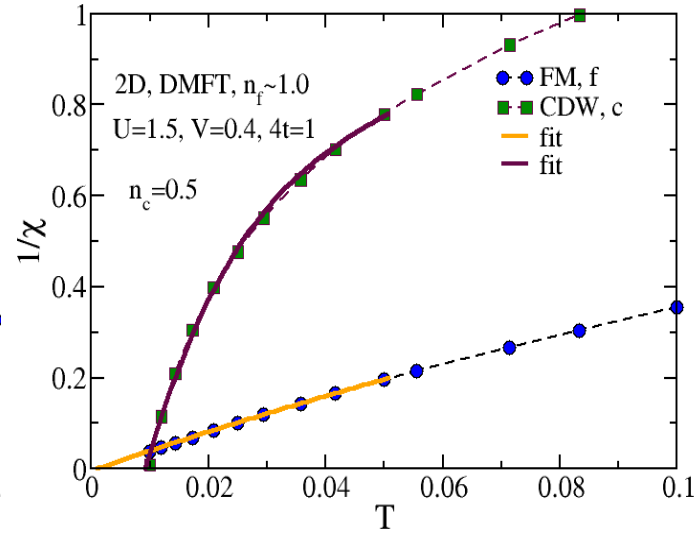
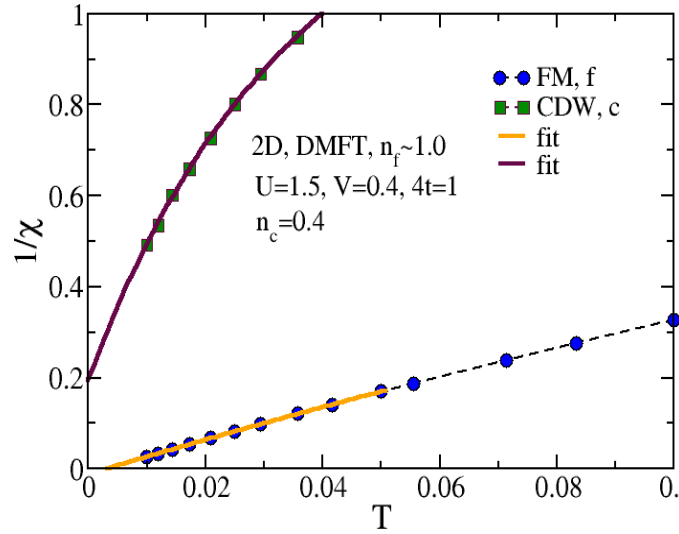
- 3D lattice shares similar phase-diagram as 2D
- Non-local correlations preserve CDW dome
- Origin??



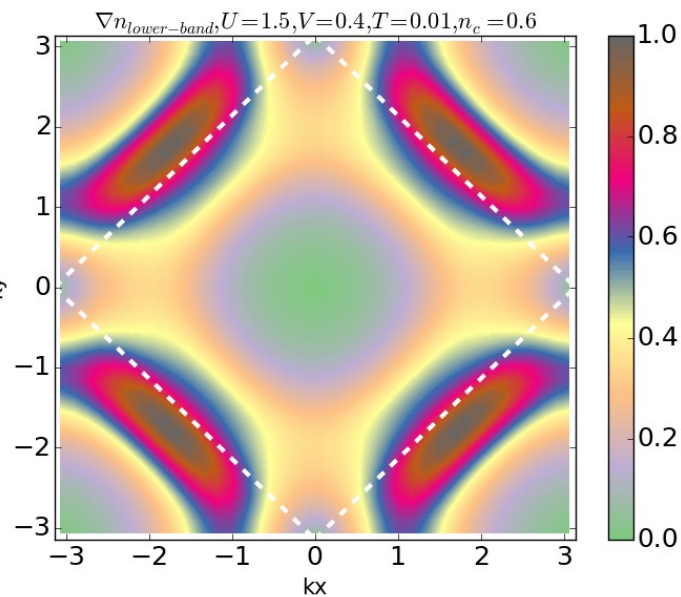
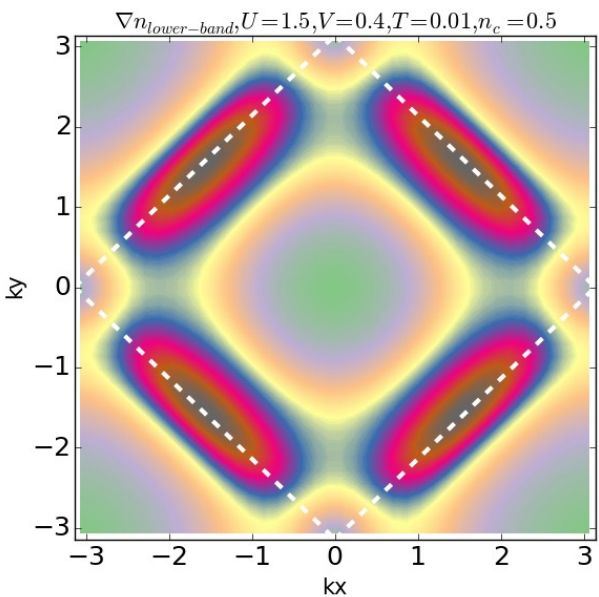
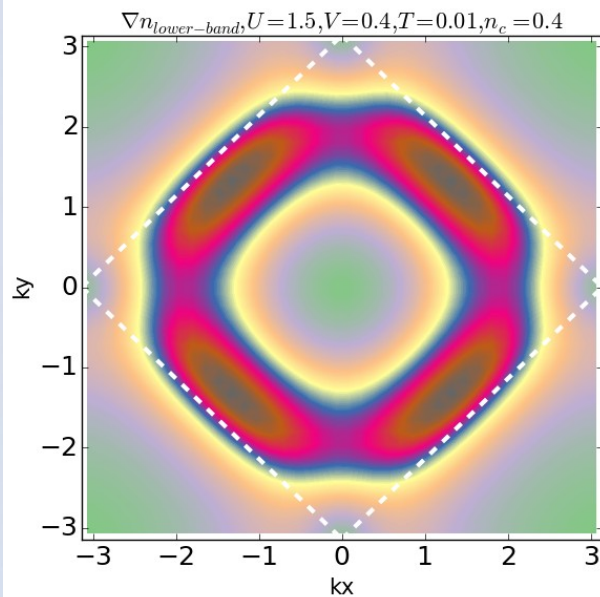
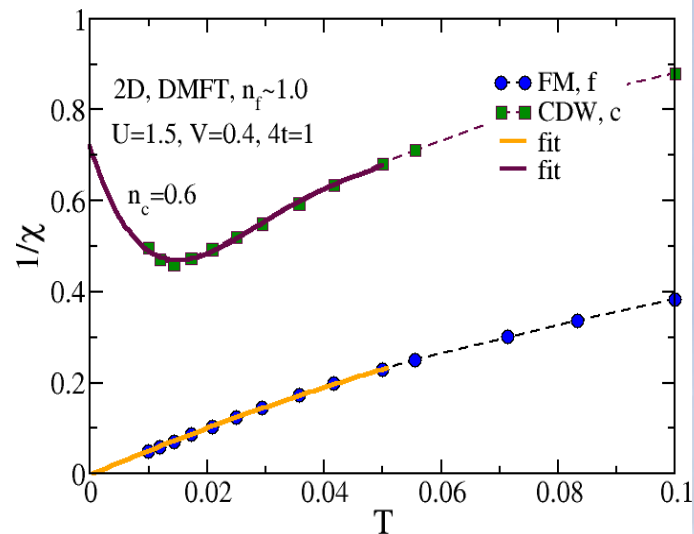
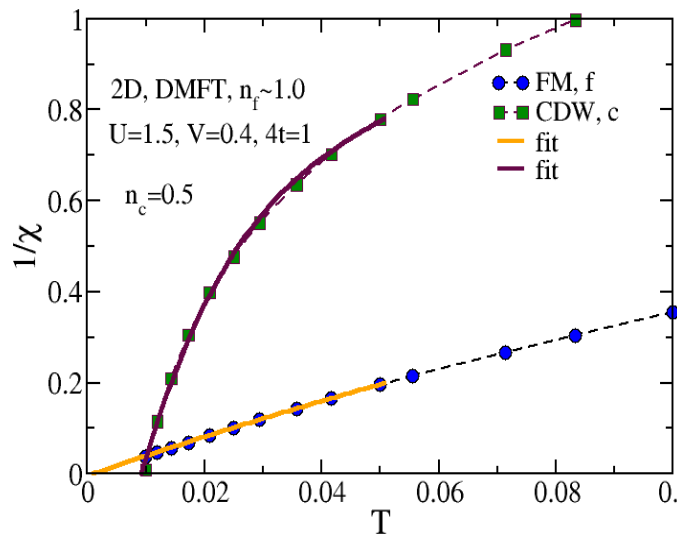
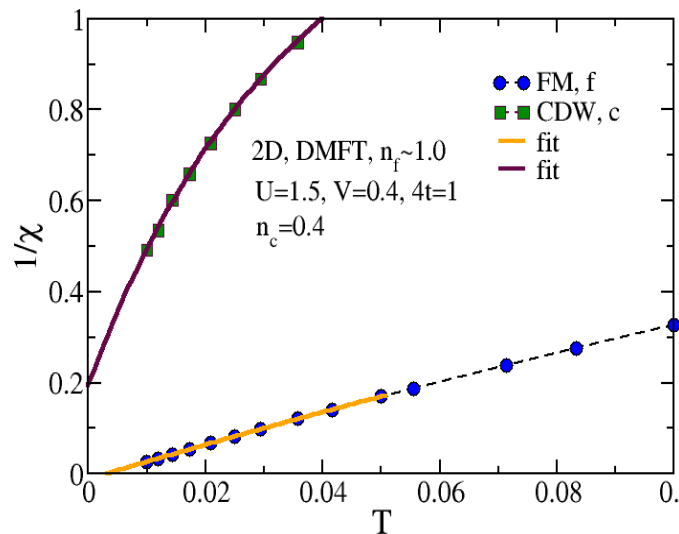
# Origin of CDW ordering



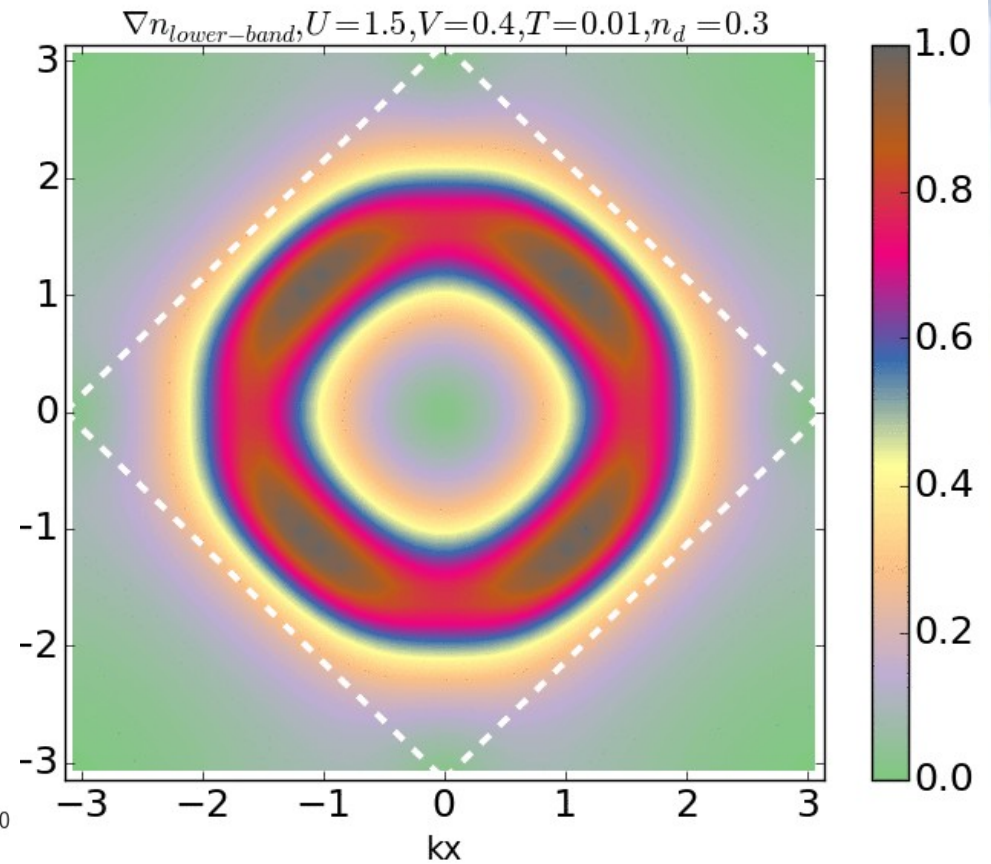
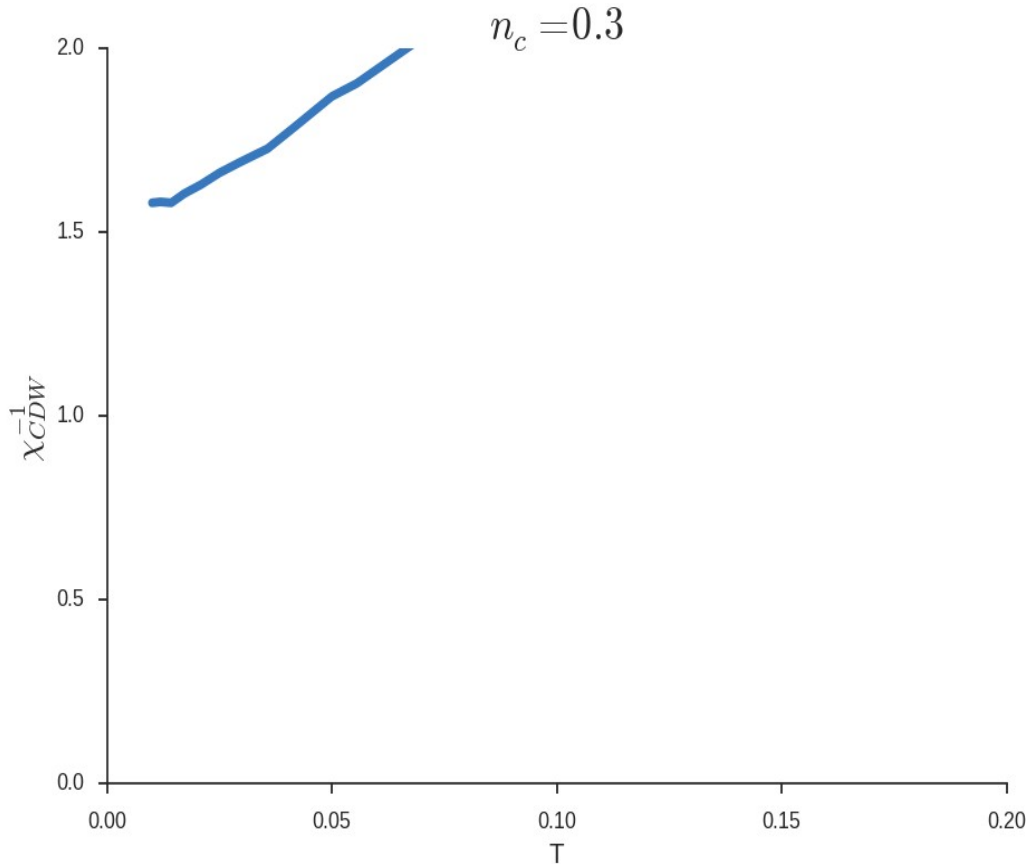
# Origin of CDW ordering



# Origin of CDW ordering

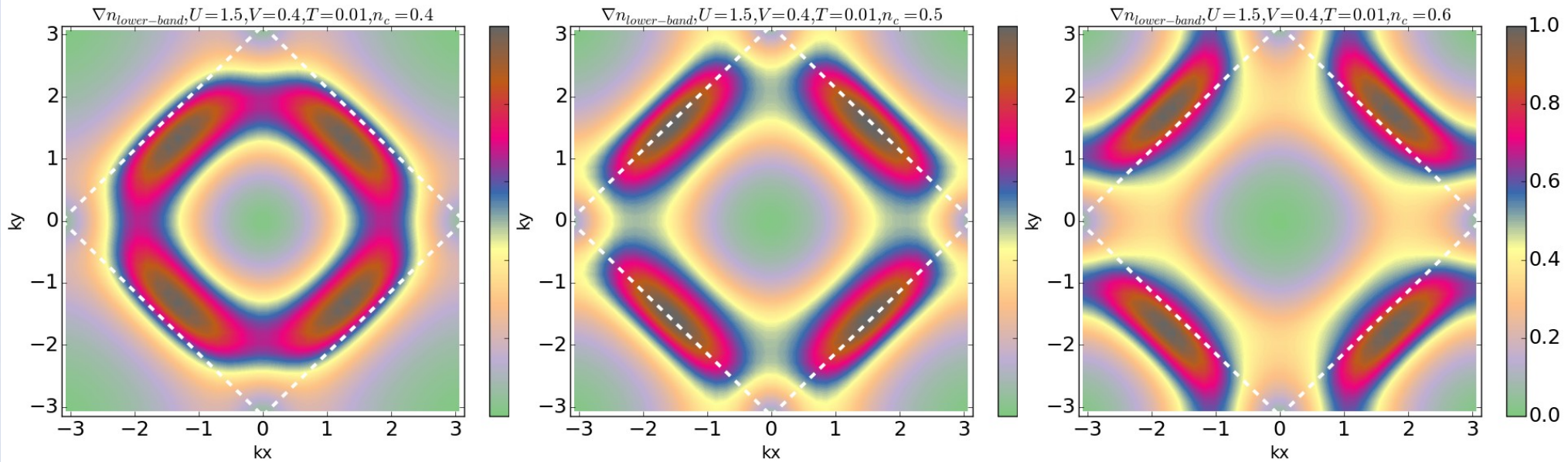


# Origin of CDW ordering



- Divergence of susceptibility happens when the Fermi surface touches on the magnetic zone boundary: perfect nesting
- Scattering is enhanced at  $q=(\pi,\pi)$  due to perfect nesting

# Origin of CDW ordering



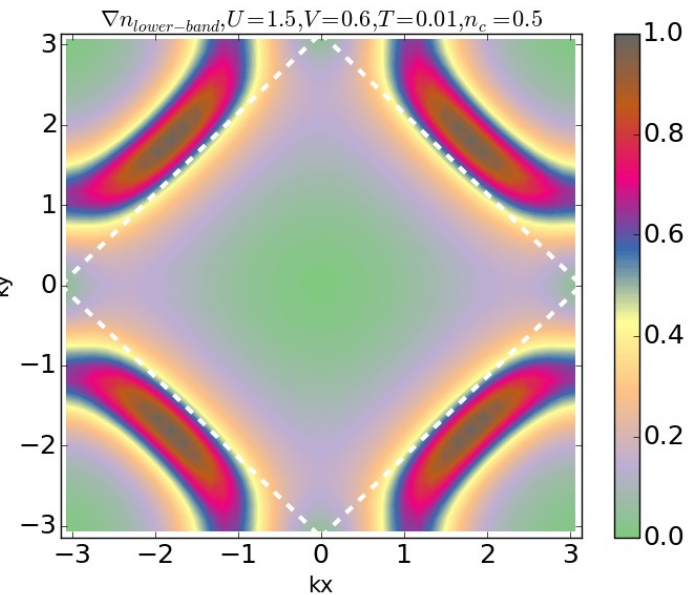
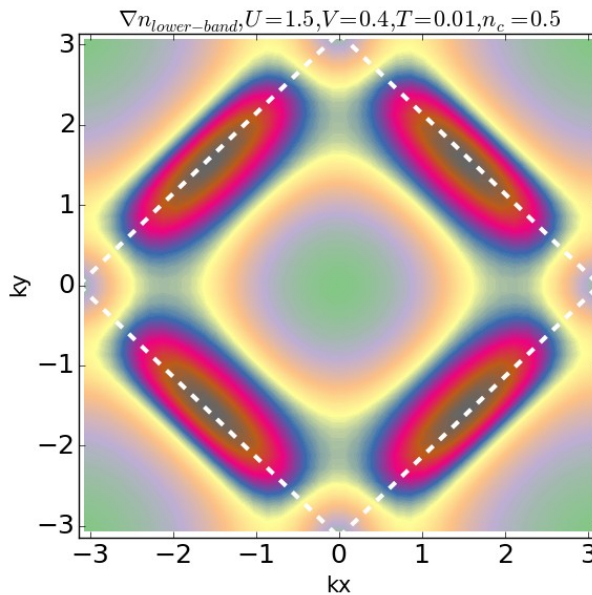
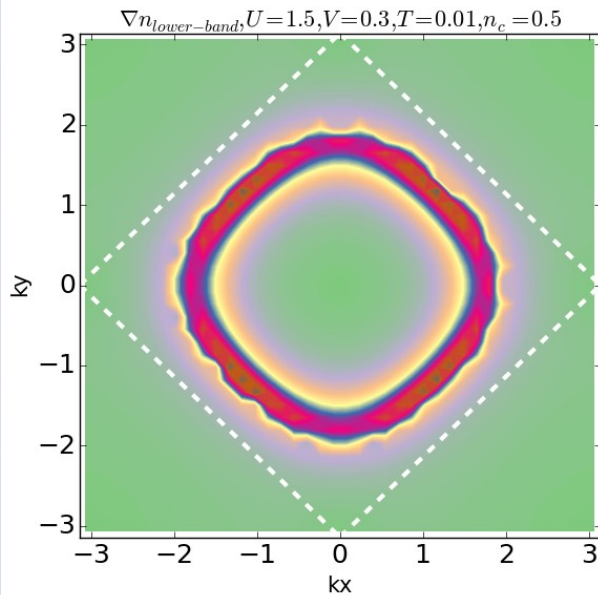
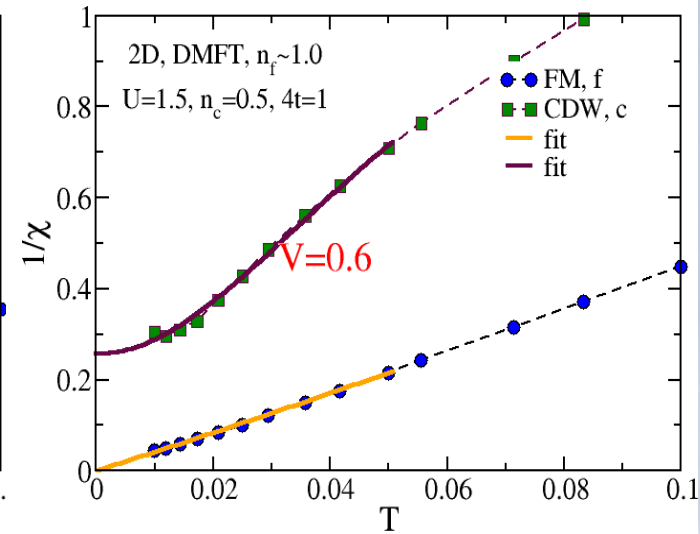
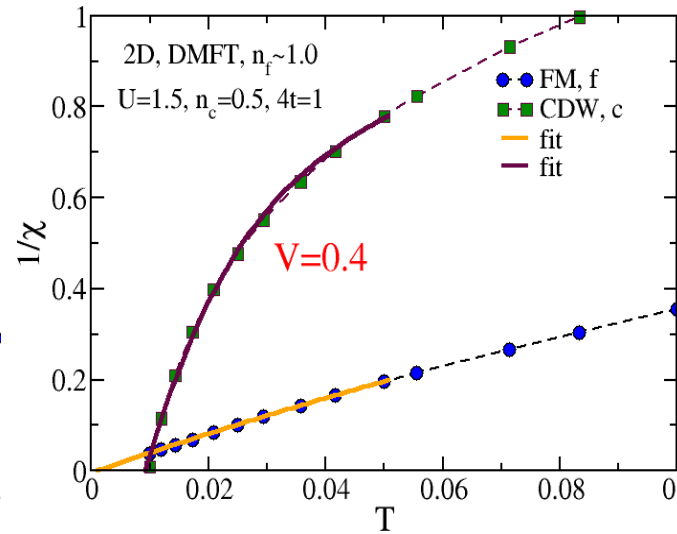
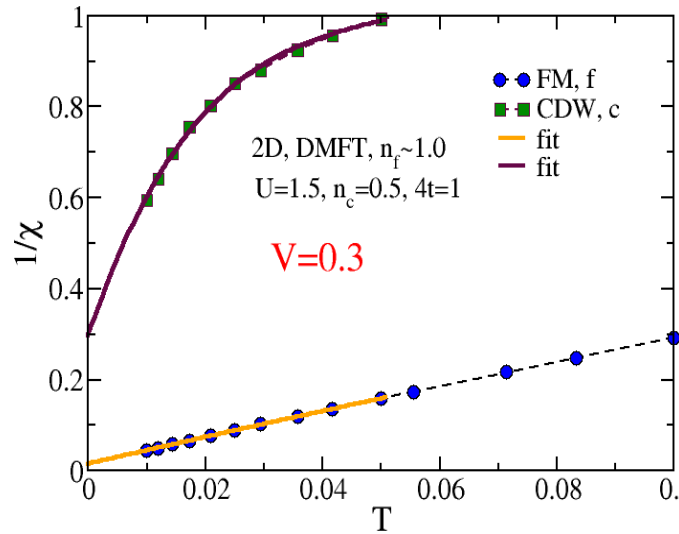
Particle-like

Perfect nesting!!

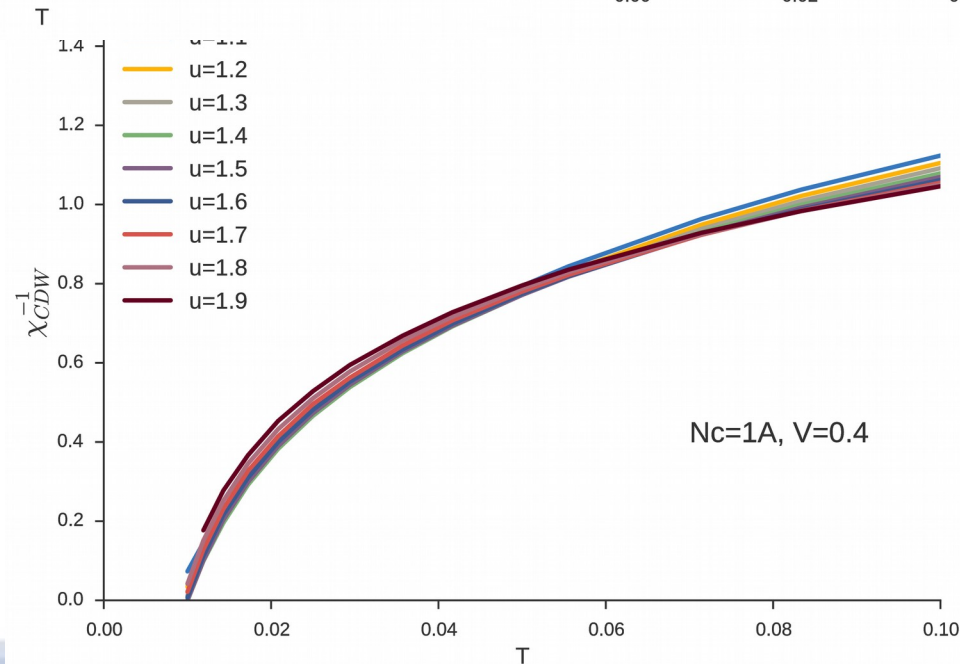
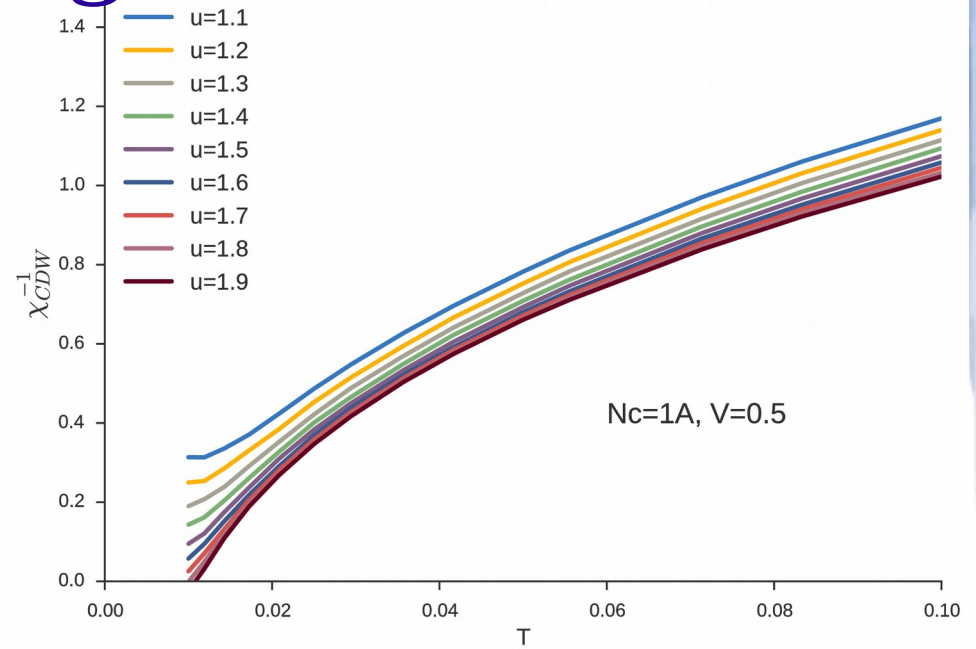
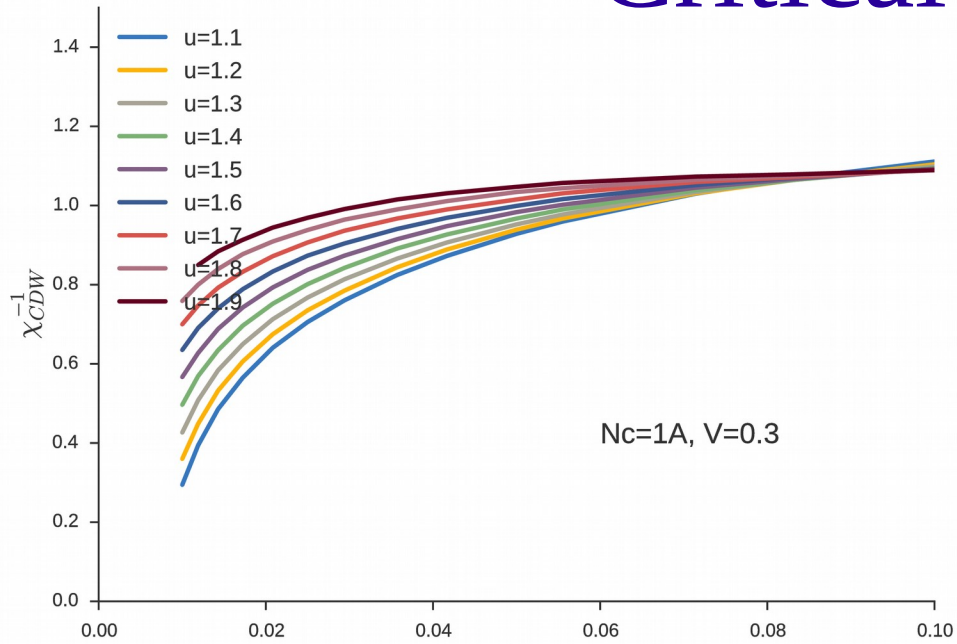
Hole-like

- The Fermi surface changes its topology from particle-like to hole-like at the transition and thus is a **Lifshitz transition**.
- Here the control parameter is the c-band filling

# Hybridization as the control parameter



# Critical region?



# Conclusion

- Using DMFT, DCA and DF, we have determined the phase-diagram of periodic Anderson model with f-band at half-filling and c-band around quarter-filling for both 2D and 3D lattices.
- A dome-shape CDW phase surrounding quarter-filling is found and persists when non-local correlations are included through DCA and DF.
- The CDW ordering is explained by the perfect nesting of the Fermi-surface of the renormalized lower-band.
- The Fermi surface changes its topology from particle-like to hole-like at the transition and thus is a Lifshitz transition.
- A critical  $V$  is found where CDW susceptibility lines collapse on a single curve.

Thank you!