

Strongly correlated superconductivity in cuprates and layered organics: results and some algorithmic details

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Bâton Rouge, 14 February 2015

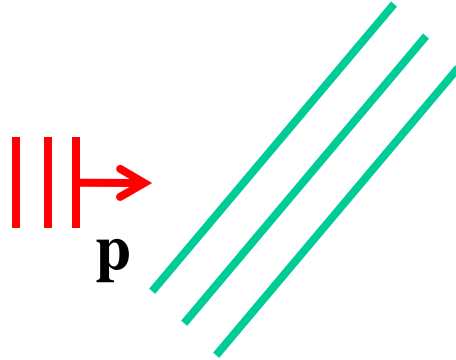


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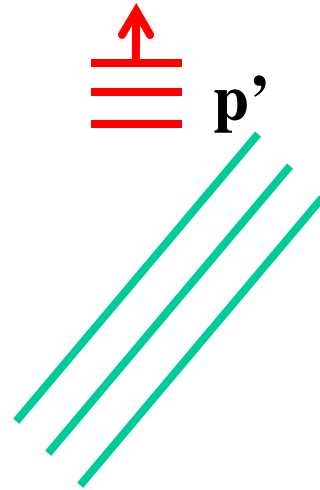
BCS Superconductivity



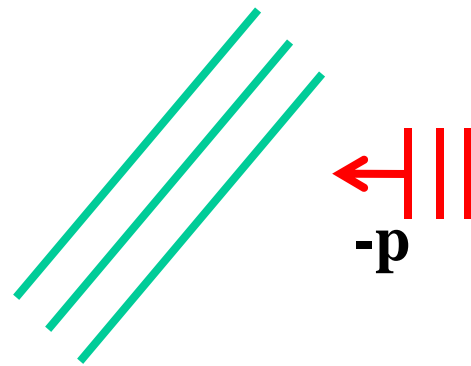
Attraction mechanism in the metallic state



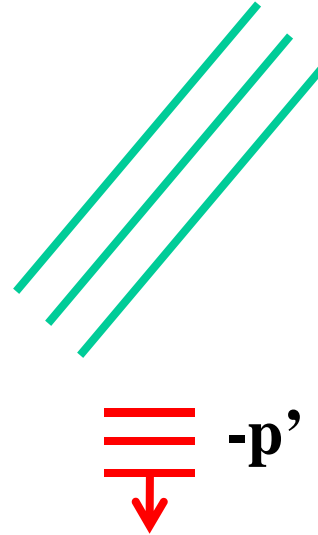
Attraction mechanism in the metallic state



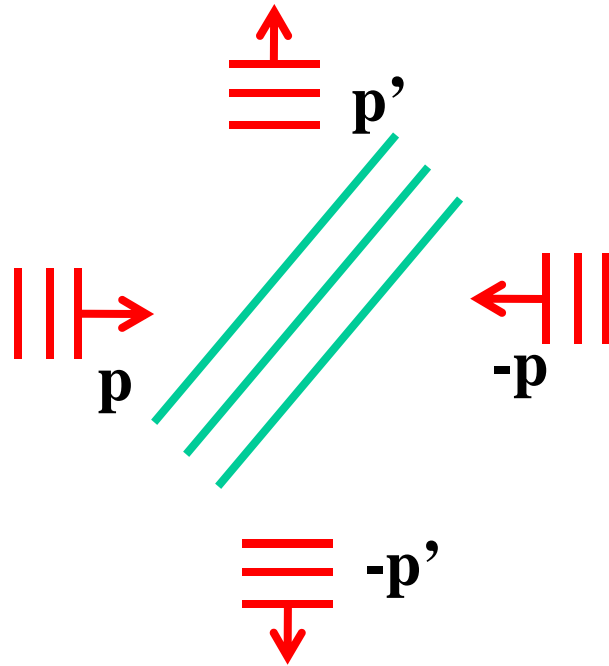
Attraction mechanism in the metallic state



Attraction mechanism in the metallic state



Attraction mechanism in the metallic state



#1 Cooper pair, #2 Phase coherence

$$E_P = \sum_{\mathbf{p}, \mathbf{p}'} U_{\mathbf{p}-\mathbf{p}'} \psi_{\mathbf{p}\uparrow, -\mathbf{p}\downarrow} \psi_{\mathbf{p}'\uparrow, -\mathbf{p}'\downarrow}^*$$

$$E_P = \sum_{\mathbf{p}, \mathbf{p}'} U_{\mathbf{p}-\mathbf{p}'} \left(\langle \psi_{\mathbf{p}\uparrow, -\mathbf{p}\downarrow} \rangle \psi_{\mathbf{p}'\uparrow, -\mathbf{p}'\downarrow}^* + \psi_{\mathbf{p}\uparrow, -\mathbf{p}\downarrow} \langle \psi_{\mathbf{p}'\uparrow, -\mathbf{p}'\downarrow}^* \rangle \right)$$

$$|\text{BCS}(\theta)\rangle = \dots + e^{iN\theta} |N\rangle + e^{i(N+2)\theta} |N+2\rangle + \dots$$



Breakdown of band theory

Half-filled band is metallic?

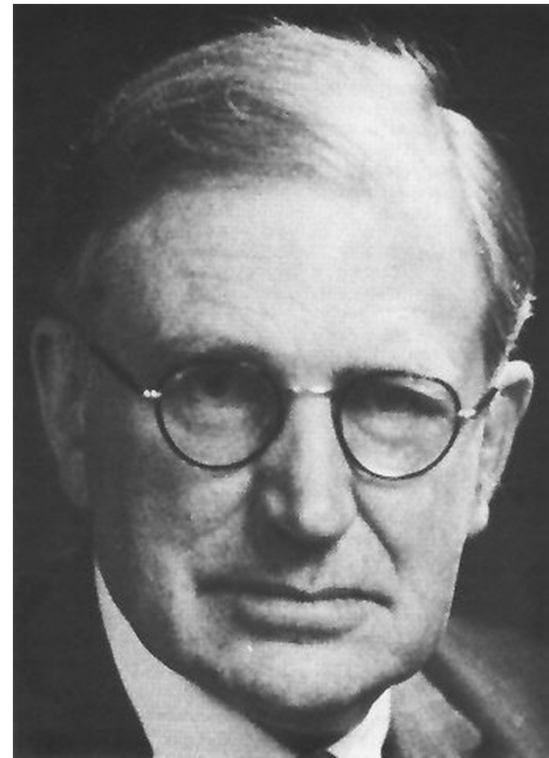


Half-filled band: Not always a metal

NiO, Boer and Verway



Peierls, 1937



Mott, 1949



Two materials,
two routes to breakdown of band theory
and of BCS superconductivity



Cuprates

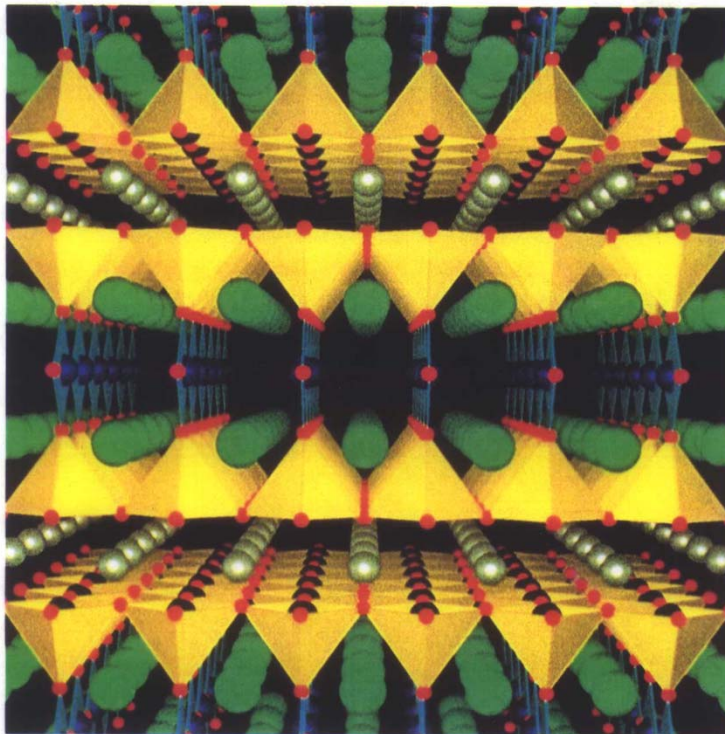
SCIENTIFIC AMERICAN

JUNE 1988
\$3.50

How nonsense is deleted from genetic messages.

R_x for economic growth: aggressive use of new technology.

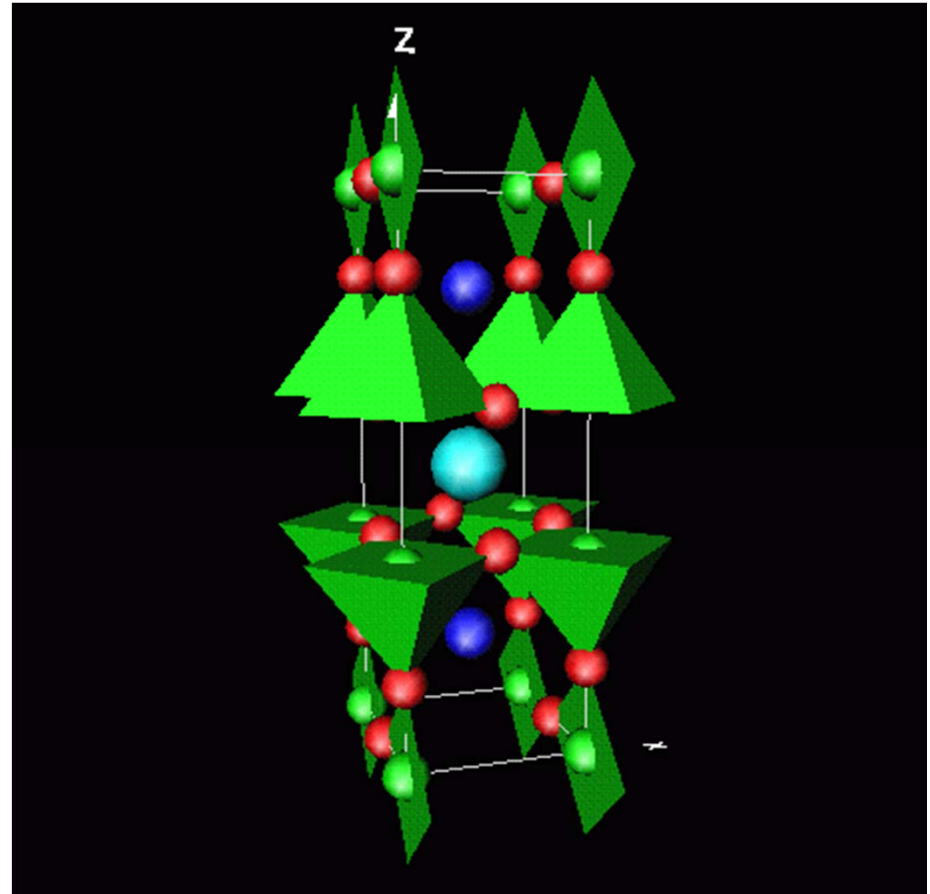
Can particle physics test cosmology?



High-Temperature Superconductor belongs to a family of materials that exhibit exotic electronic properties.

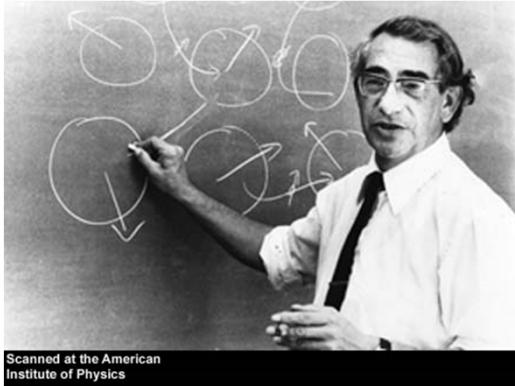


92-37

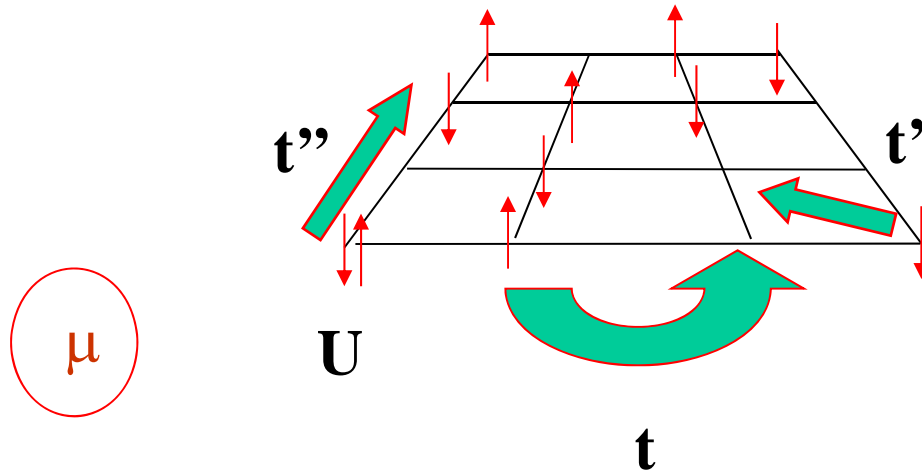


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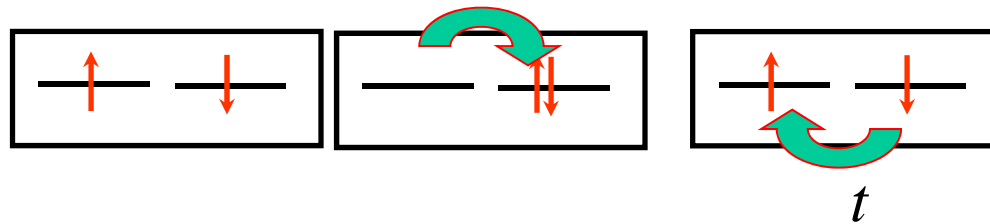
Hubbard model



1931-1980



$$H = - \sum_{\langle ij \rangle \sigma} t_{i,j} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



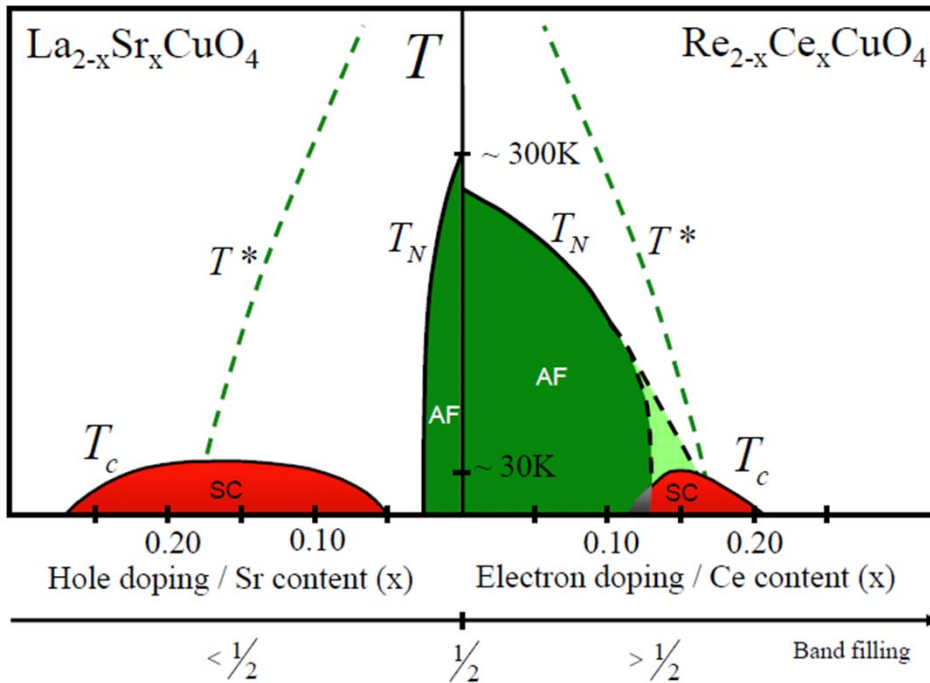
$t = 1$

Effective model, Heisenberg: $J = 4t^2 / U$



High-temperature superconductors

Armitage, Fournier, Greene, RMP (2009)



- Competing order

- Current loops: Varma, PRB **81**, 064515 (2010)
- Stripes or nematic: Kivelson et al. RMP **75** 1201(2003); J.C.Davis
- d-density wave : Chakravarty, Nayak, Phys. Rev. B **63**, 094503 (2001); Affleck et al. flux phase
- SDW: Sachdev PRB **80**, 155129 (2009) ...

- Or Mott Physics?

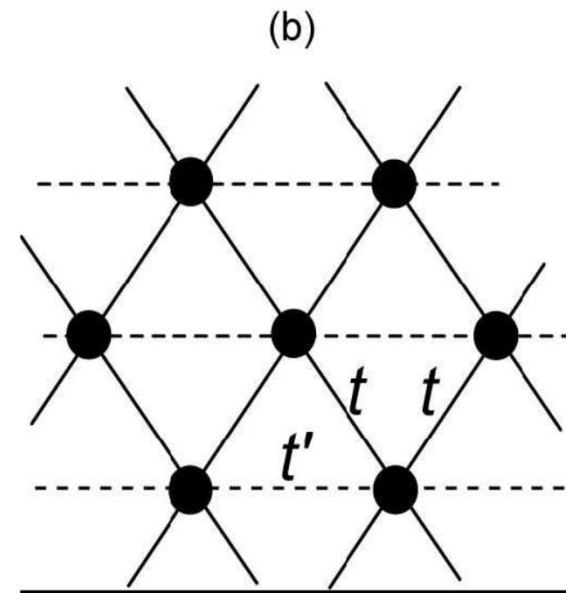
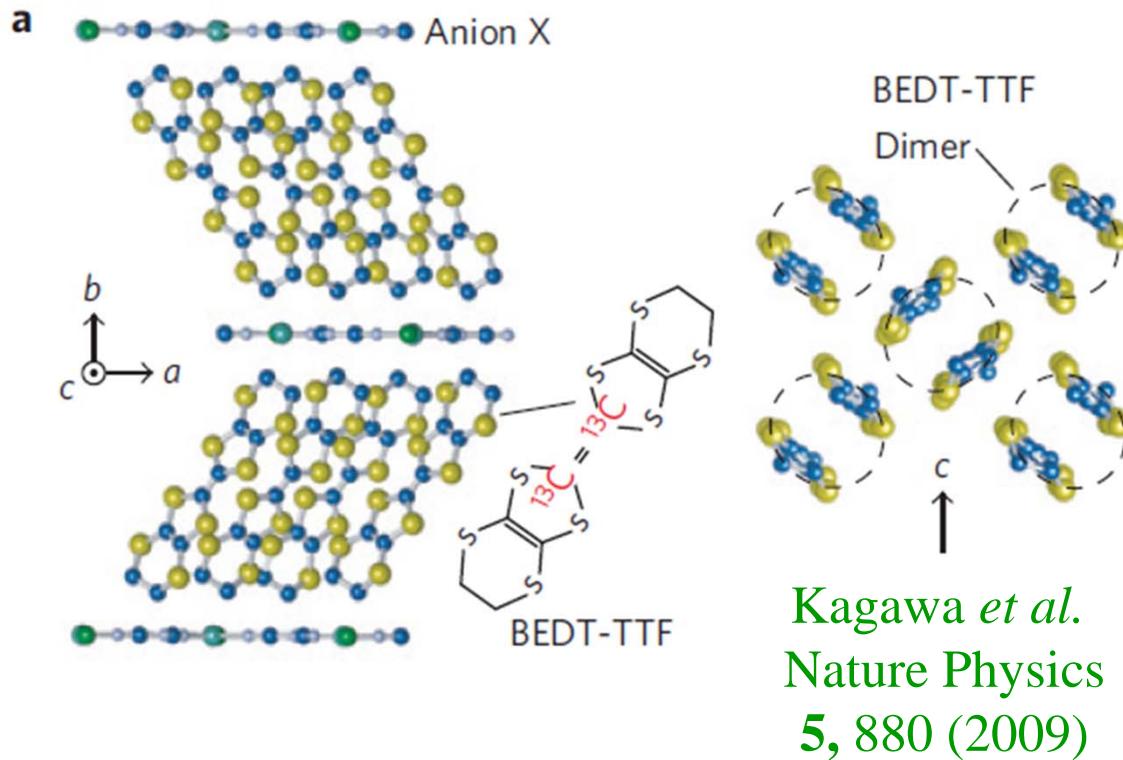
- RVB: P.A. Lee Rep. Prog. Phys. **71**, 012501 (2008)

What is under the dome?
Mott Physics away from $n = 1$



Hubbard on anisotropic triangular lattice

H. Kino + H. Fukuyama, J. Phys. Soc. Jpn **65** 2158 (1996),
 R.H. McKenzie, Comments Condens Mat Phys. **18**, 309 (1998)



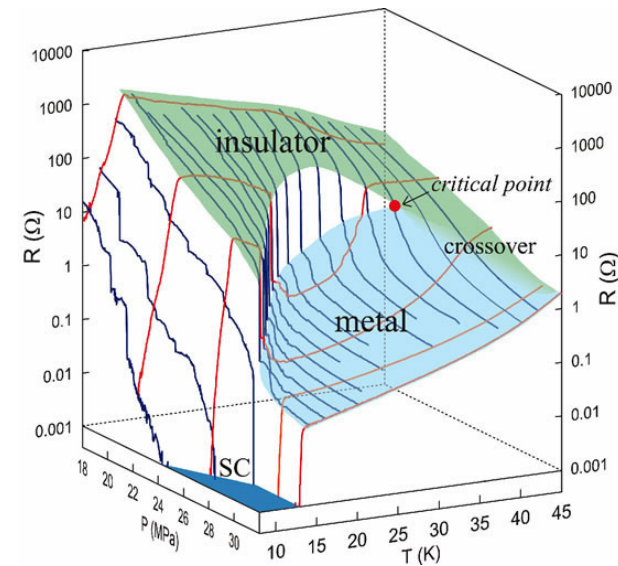
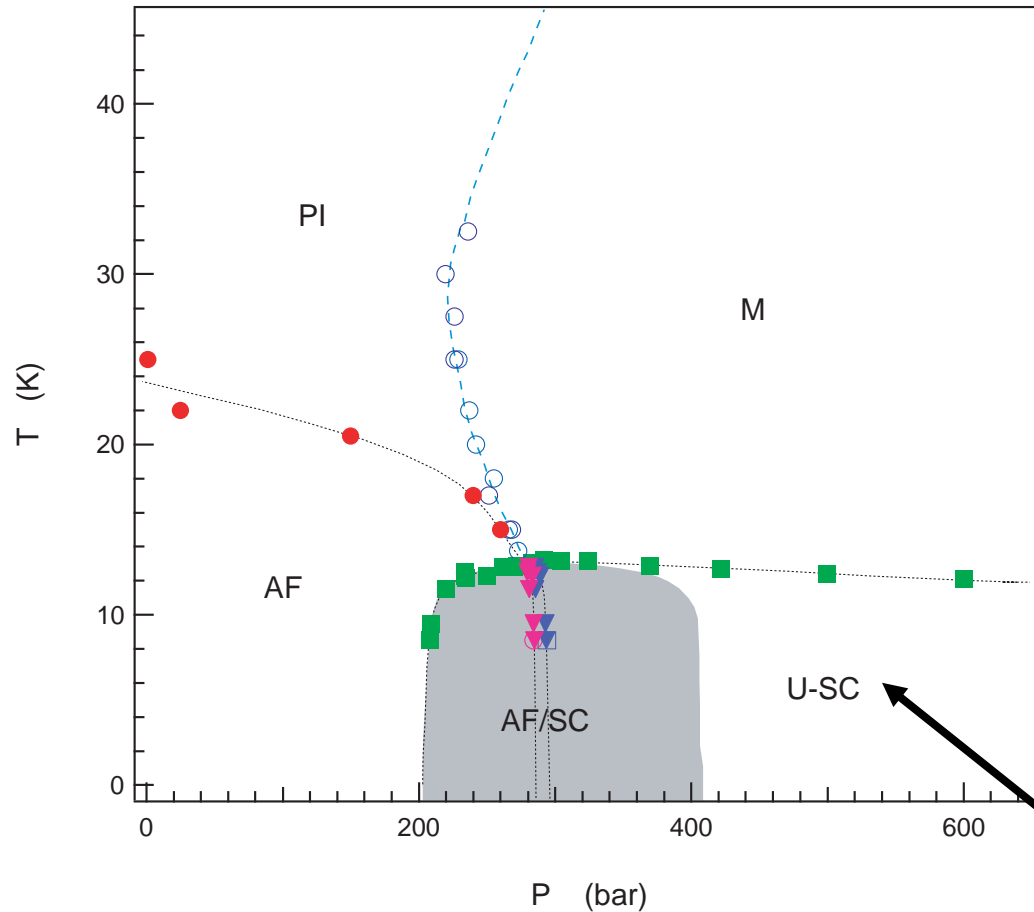
$$t \approx 50 \text{ meV}$$

$$\Rightarrow U \approx 400 \text{ meV}$$

$$t'/t \sim 0.6 - 1.1$$

$$H = \sum_{ij\sigma} (t_{ij} - \delta_{ij}\mu) c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Phase diagram for organics



F. Kagawa, K. Miyagawa, + K. Kanoda
PRB **69** (2004) + Nature **436** (2005)

B_g for C_{2h} and B_{2g} for D_{2h}

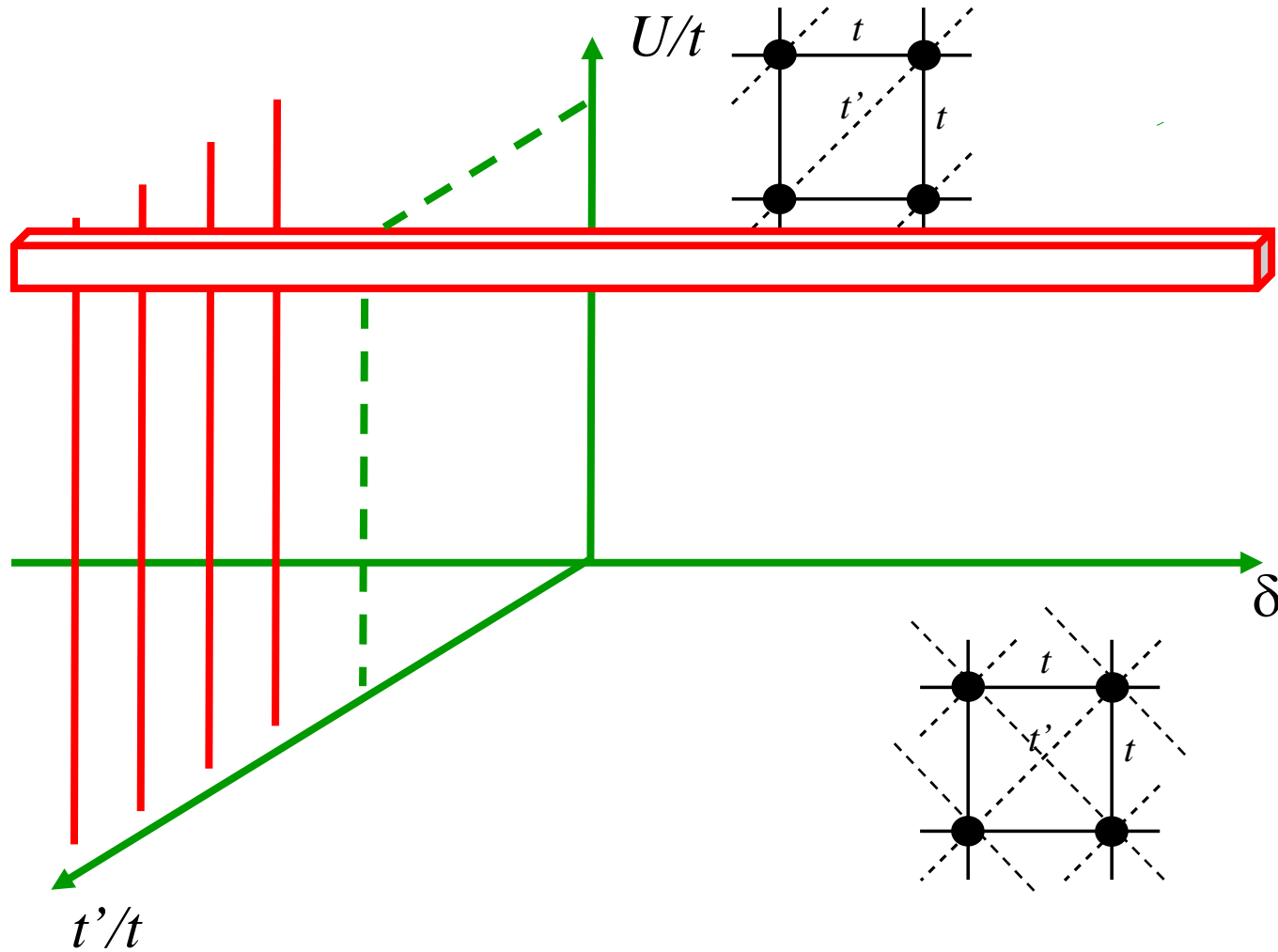
Powell, McKenzie cond-mat/0607078

Phase diagram ($X=Cu[N(CN)_2]Cl$)

S. Lefebvre et al. PRL **85**, 5420 (2000), P. Limelette, et al. PRL **91** (2003)



Perspective

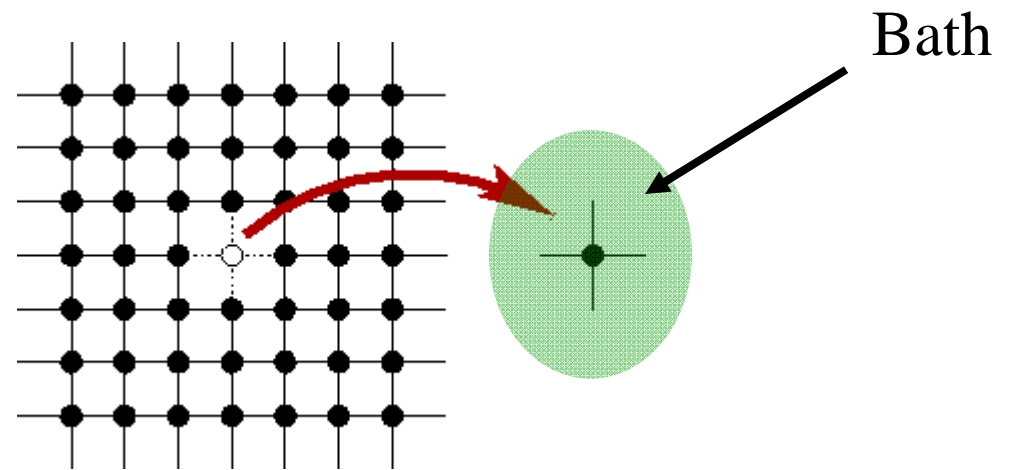


Methods

Mott transition and Dynamical Mean-Field Theory.

The beginnings in $d = \text{infinity}$

- Compute scattering rate (self-energy) of impurity problem.
- Use that self-energy (ω dependent) for lattice.
- Project lattice on single-site and adjust bath so that single-site DOS obtained both ways be equal.

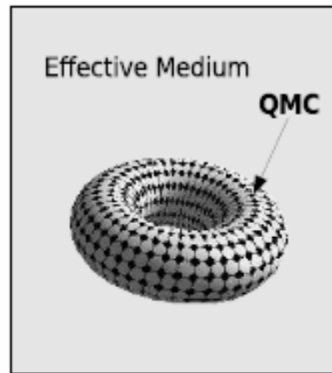


W. Metzner and D. Vollhardt, PRL (1989)
A. Georges and G. Kotliar, PRB (1992)
M. Jarrell PRB (1992)

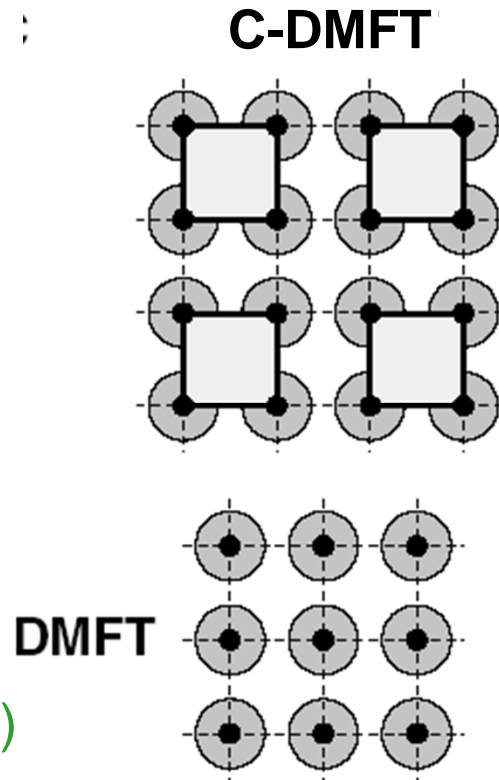
DMFT, ($d = 3$)



2d Hubbard: Quantum cluster method



DCA



Hettler ...Jarrell...Krishnamurty PRB **58** (1998)

Kotliar et al. PRL **87** (2001)

M. Potthoff *et al.* PRL **91**, 206402 (2003).

REVIEWS

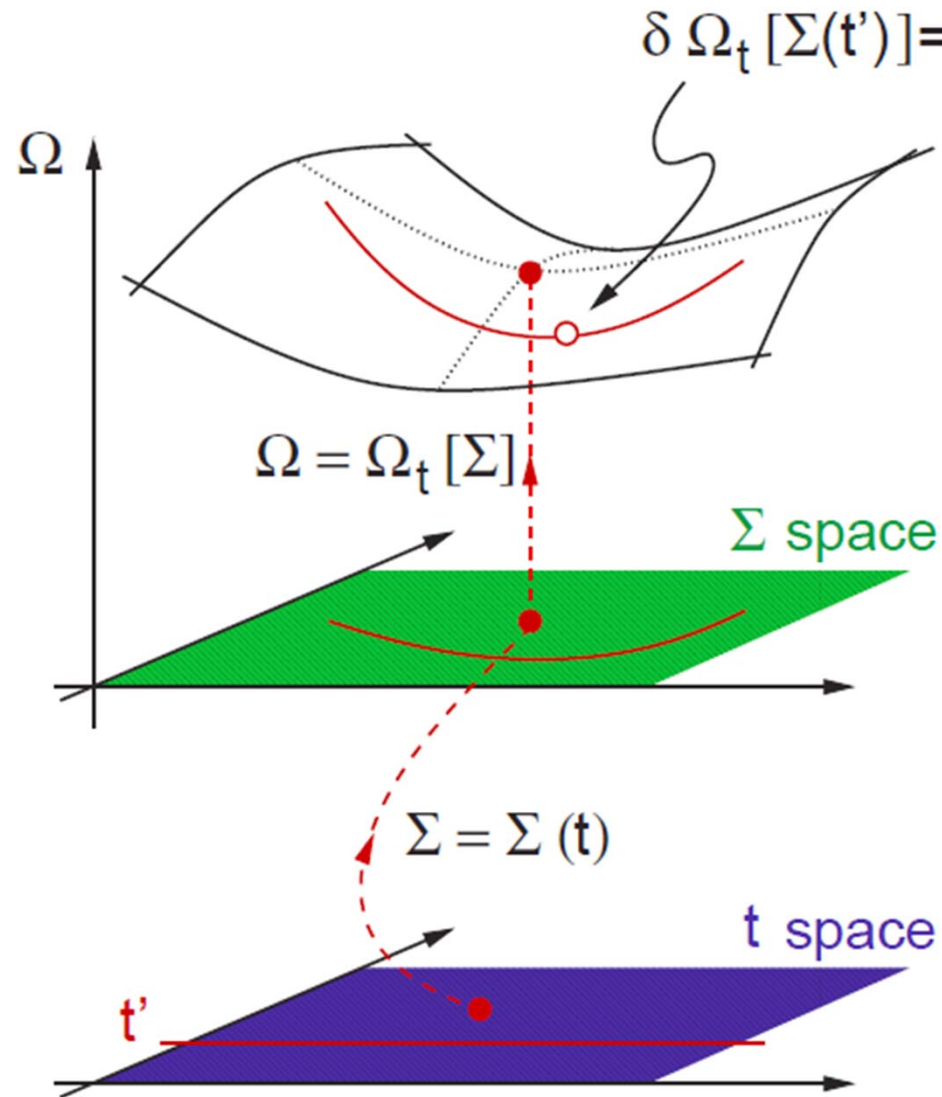
Maier, Jarrell et al., RMP. (2005)

Kotliar *et al.* RMP (2006)

AMST *et al.* LTP (2006)



DMFT as a stationary point



M. Potthoff, Eur. Phys. J. B 32, 429 (2003).

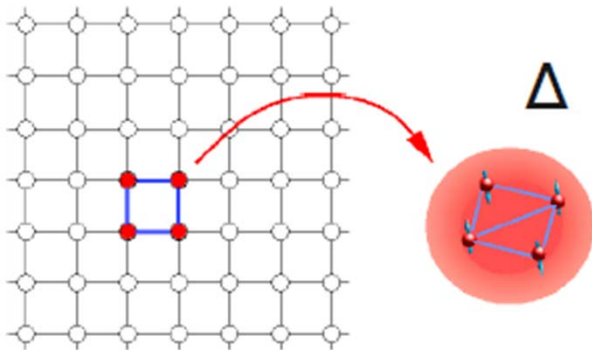


+ and -

- Long range order:
 - Allow symmetry breaking in the bath (mean-field)
- Included:
 - Short-range dynamical and spatial correlations
- Missing:
 - Long wavelength p-h and p-p fluctuations



C-DMFT

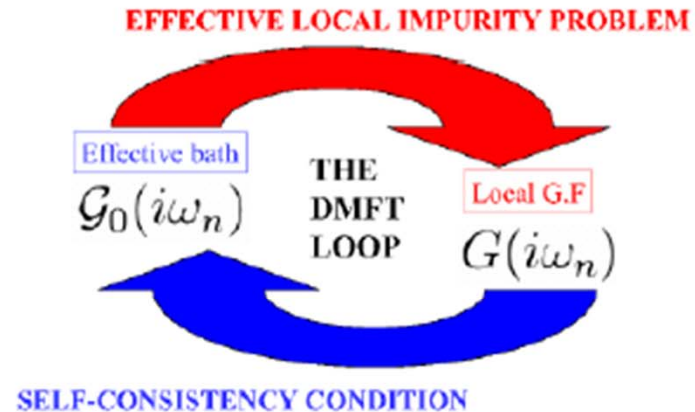


Mean-field is not a trivial problem! Many impurity solvers.

Here: continuous time QMC

P. Werner, PRL 2006
 P. Werner, PRB 2007
 K. Haule, PRB 2007

$$Z = \int \mathcal{D}[\psi^\dagger, \psi] e^{-S_c - \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{\mathbf{k}} \psi_{\mathbf{k}}^\dagger(\tau) \Delta_{\mathbf{k}}(\tau, \tau') \psi_{\mathbf{k}}(\tau')}$$



$$\Delta(i\omega_n) = i\omega_n + \mu - \Sigma_c(i\omega_n)$$

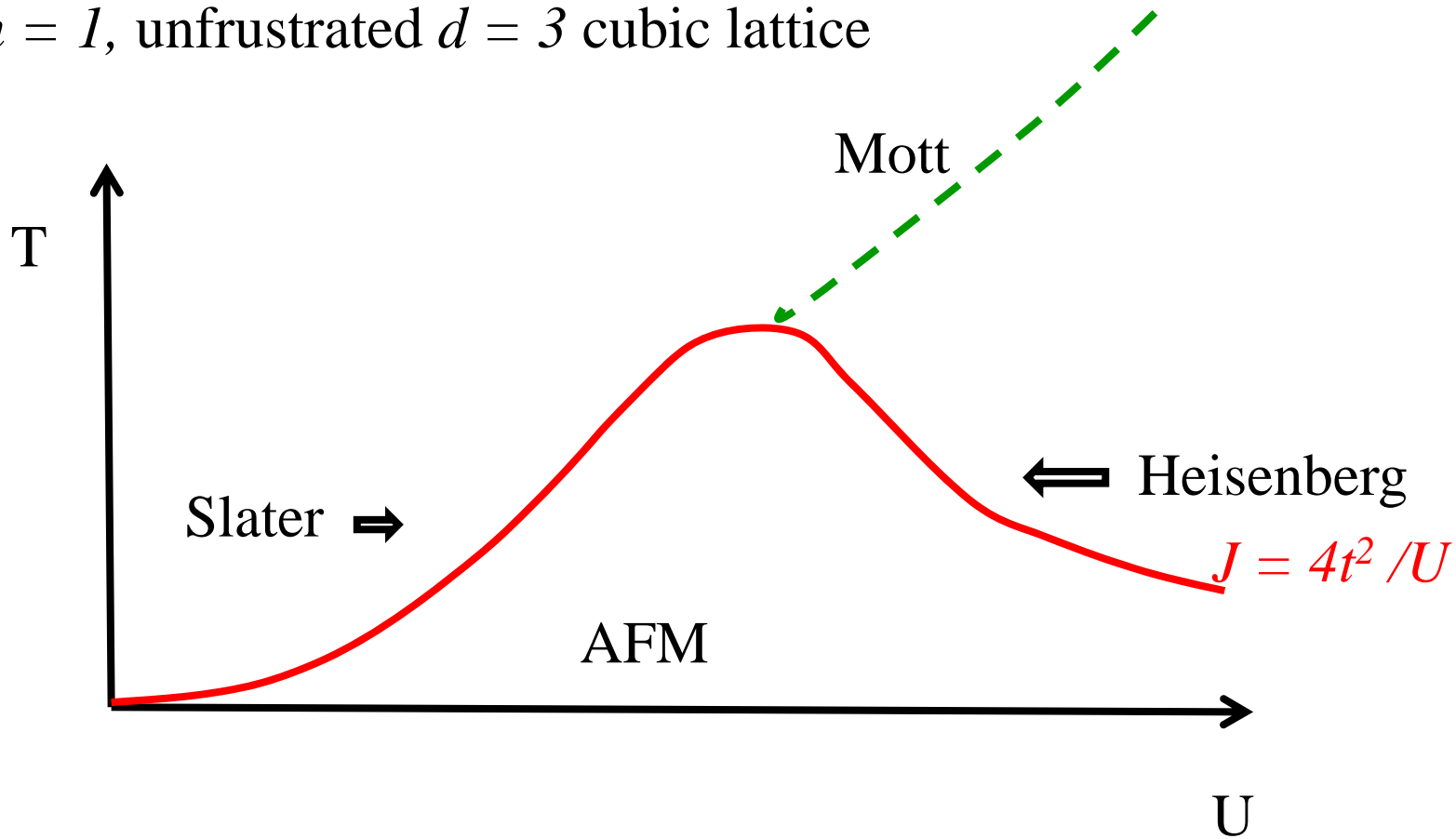
$$- \left[\sum_{\tilde{\mathbf{k}}} \frac{1}{i\omega_n + \mu - t_c(\tilde{\mathbf{k}}) - \Sigma_c(i\omega_n)} \right]^{-1}$$

Mott transition



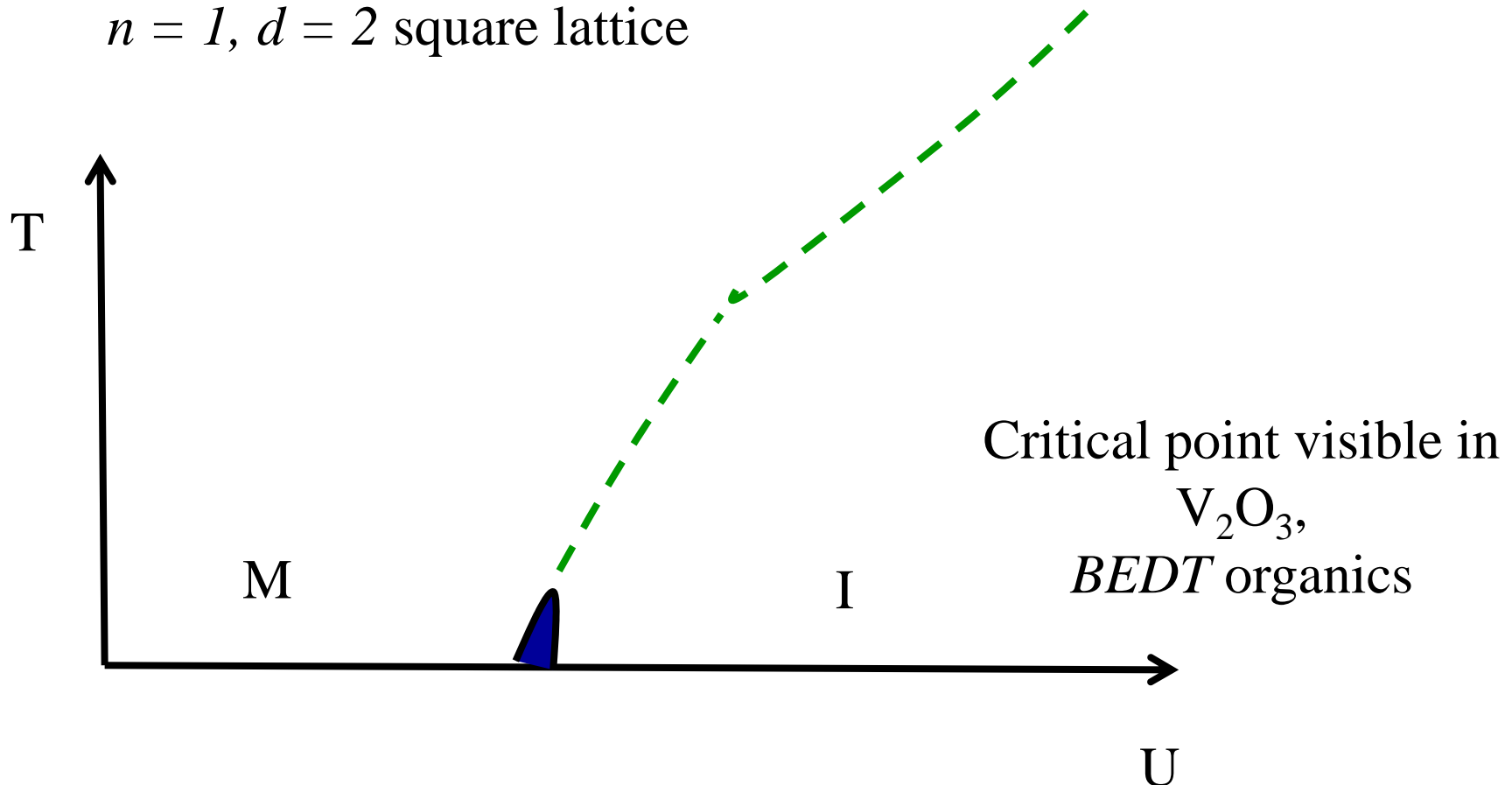
Weak vs Strong correlations

$n = 1$, unfrustrated $d = 3$ cubic lattice



Local moment and Mott transition

$n = 1, d = 2$ square lattice

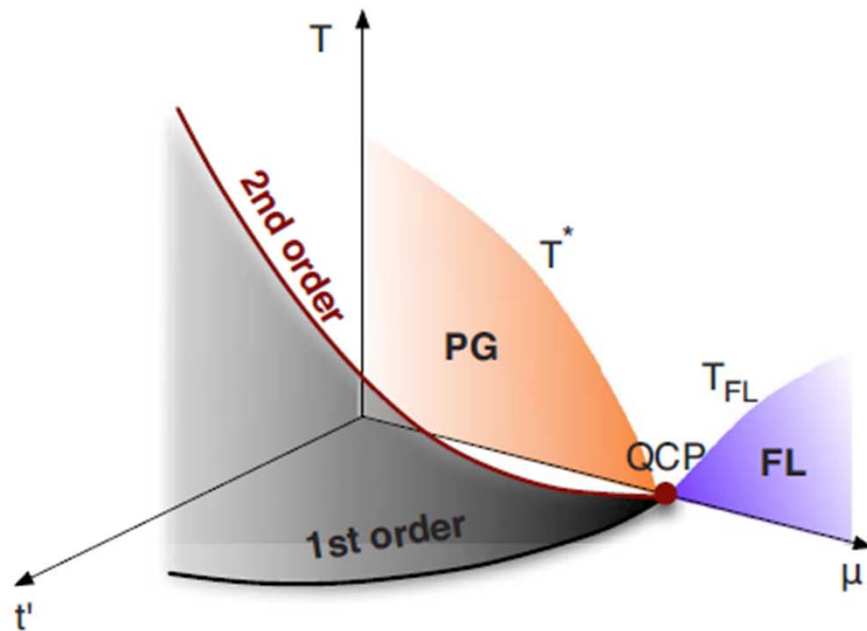


Understanding finite temperature phase from a *mean-field theory* down to $T = 0$



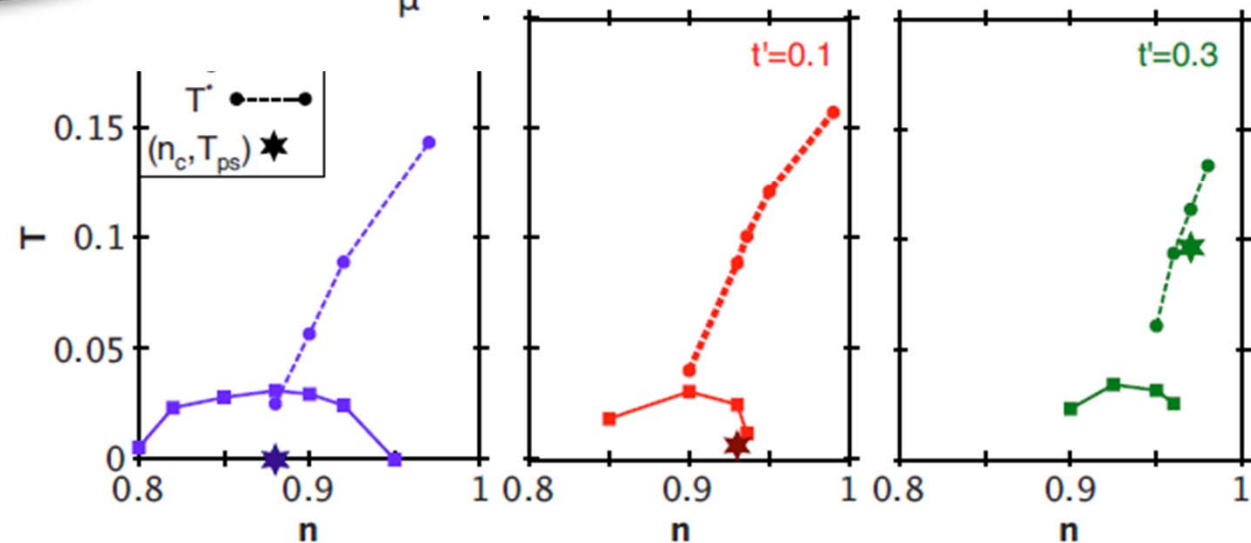
Doping

Phase separation on electron-doped side

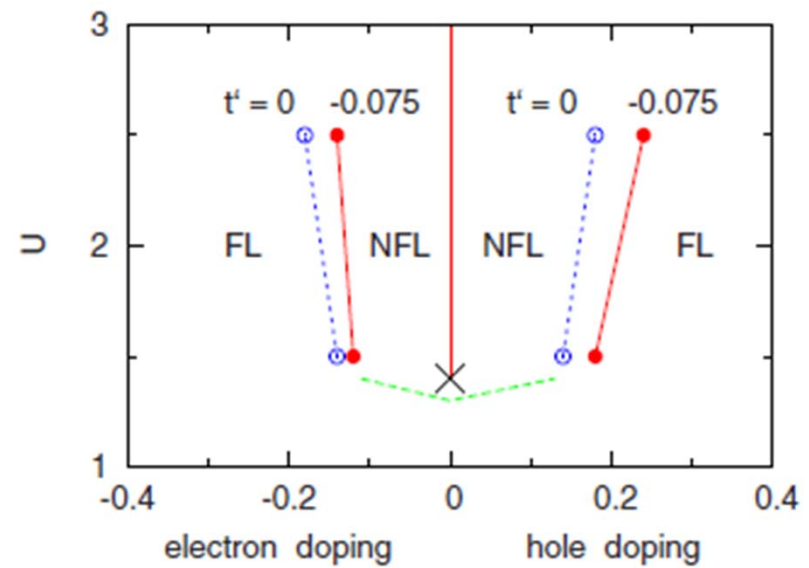
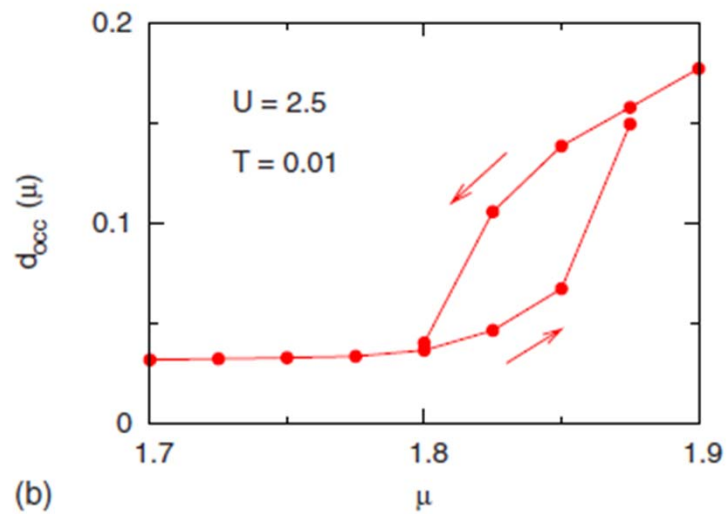


$$U=8, N_c = 8, DCA$$

E. Khatami,
 K. Mielson,
 D. Galanakis,
 A. Macridin,
 J. Moreno,
 R. T. Scalettar, and
 M. Jarrell
 PRB **81**, 201101(R)
 2010



Crossovers and transition

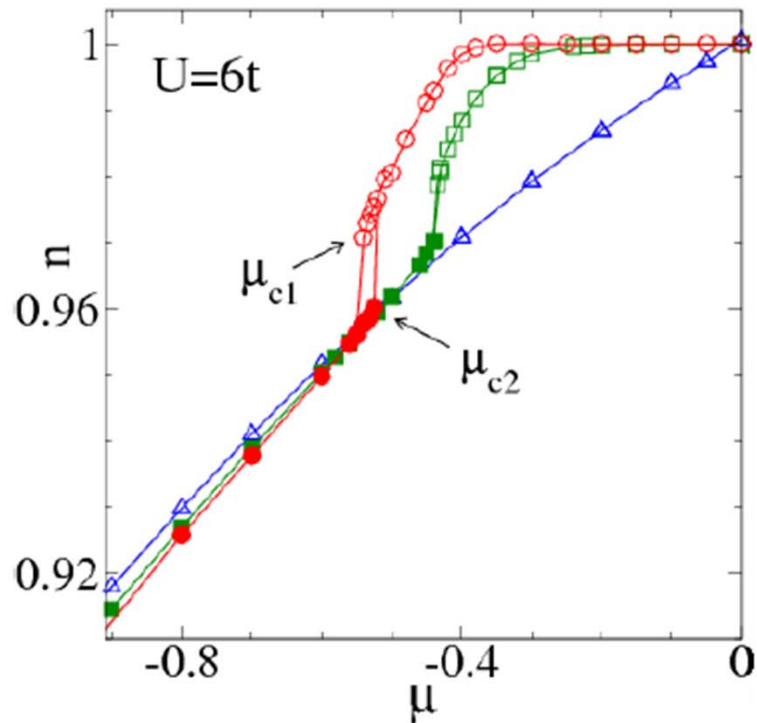


A. Liebsch, N.H. Tong, PRB **80**, 165126 (2009)



First order transition at finite doping

$$t' = 0$$

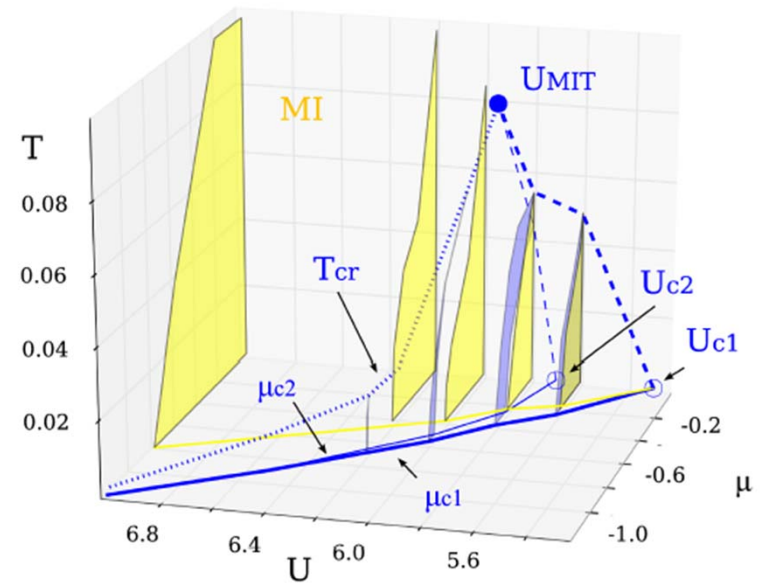
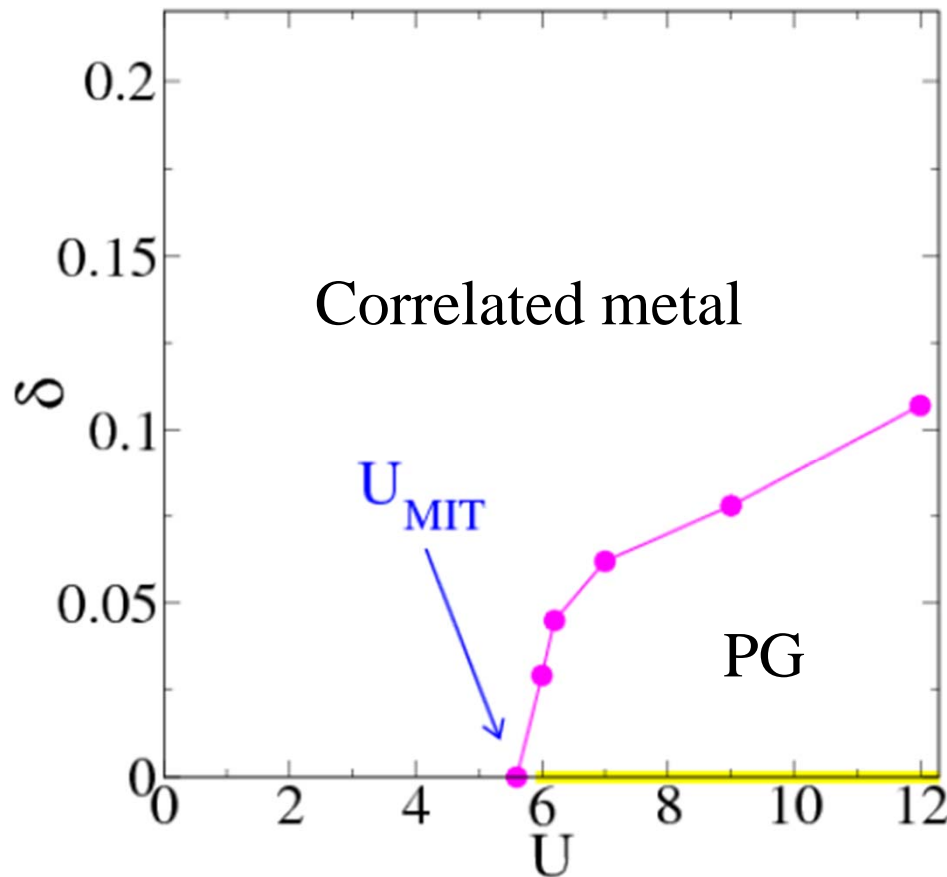


$n(\mu)$ for several temperatures:
 $T/t = 1/10, 1/25, 1/50$

Sordi et al. PRL 2010, PRB 2011

A finite-doping first order transition, linked to Mott transition up to optimal doping

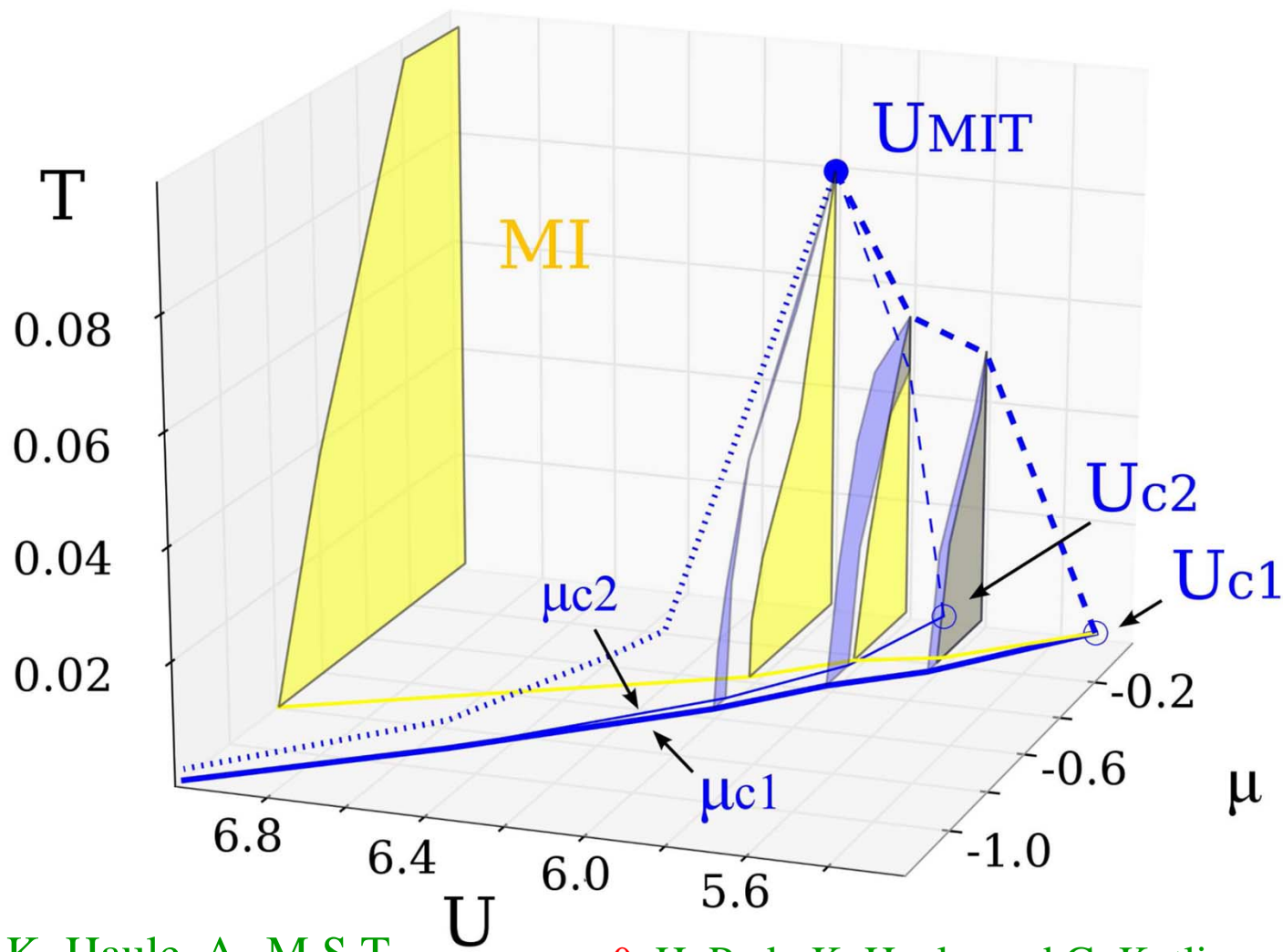
Doping dependence of critical point as a function of U



Sordi et al. PRL 2010, PRB 2011



Normal state phase diagram



G. Sordi, K. Haule, A.-M.S.T
PRL, **104**, 226402 (2010)

$\mu = 0$, H. Park, K. Haule, and G. Kotliar,
Phys. Rev. Lett. 101, 186403 (2008)



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Giovanni Sordi



Patrick Sémon



Kristjan Haule

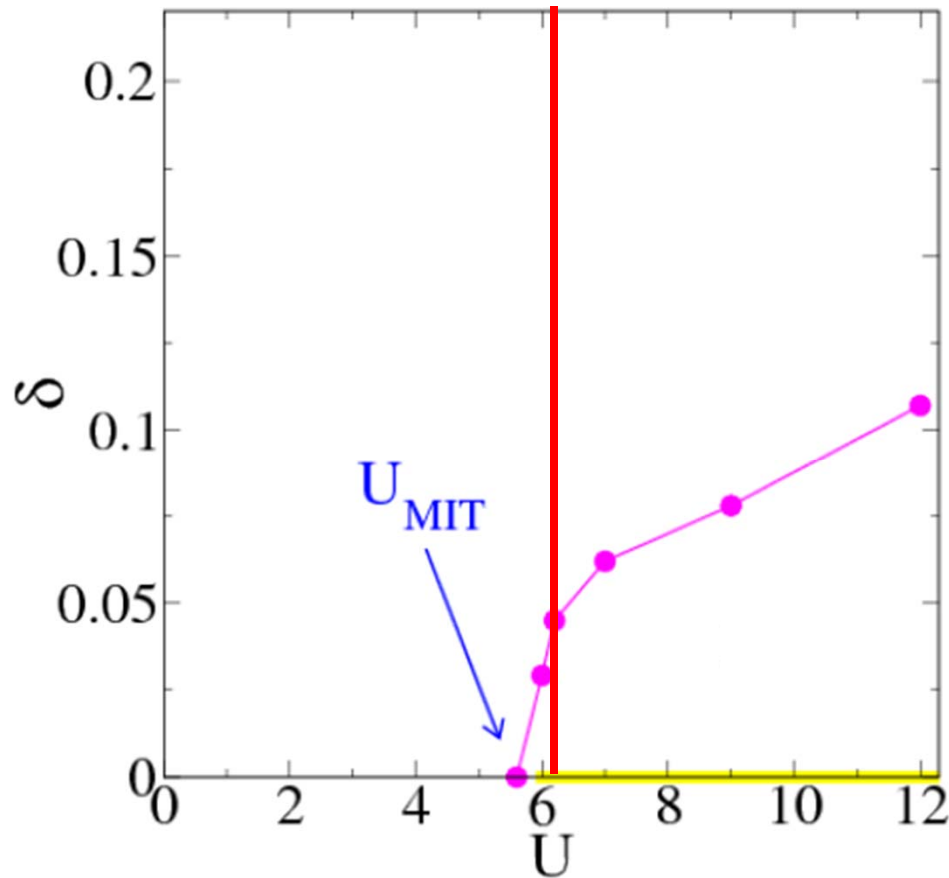
The Wisdom line

G. Sordi, *et al.* Scientific Reports 2, 547 (2012)

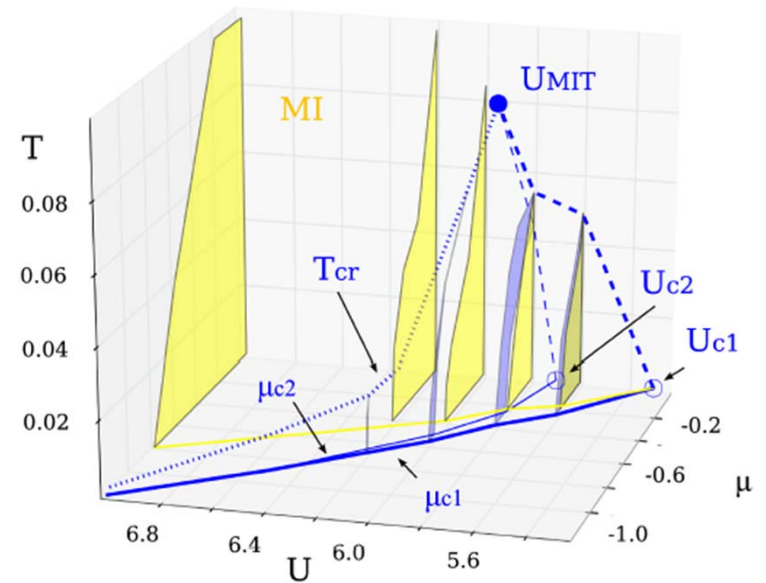


Link to Mott transition up to optimal doping

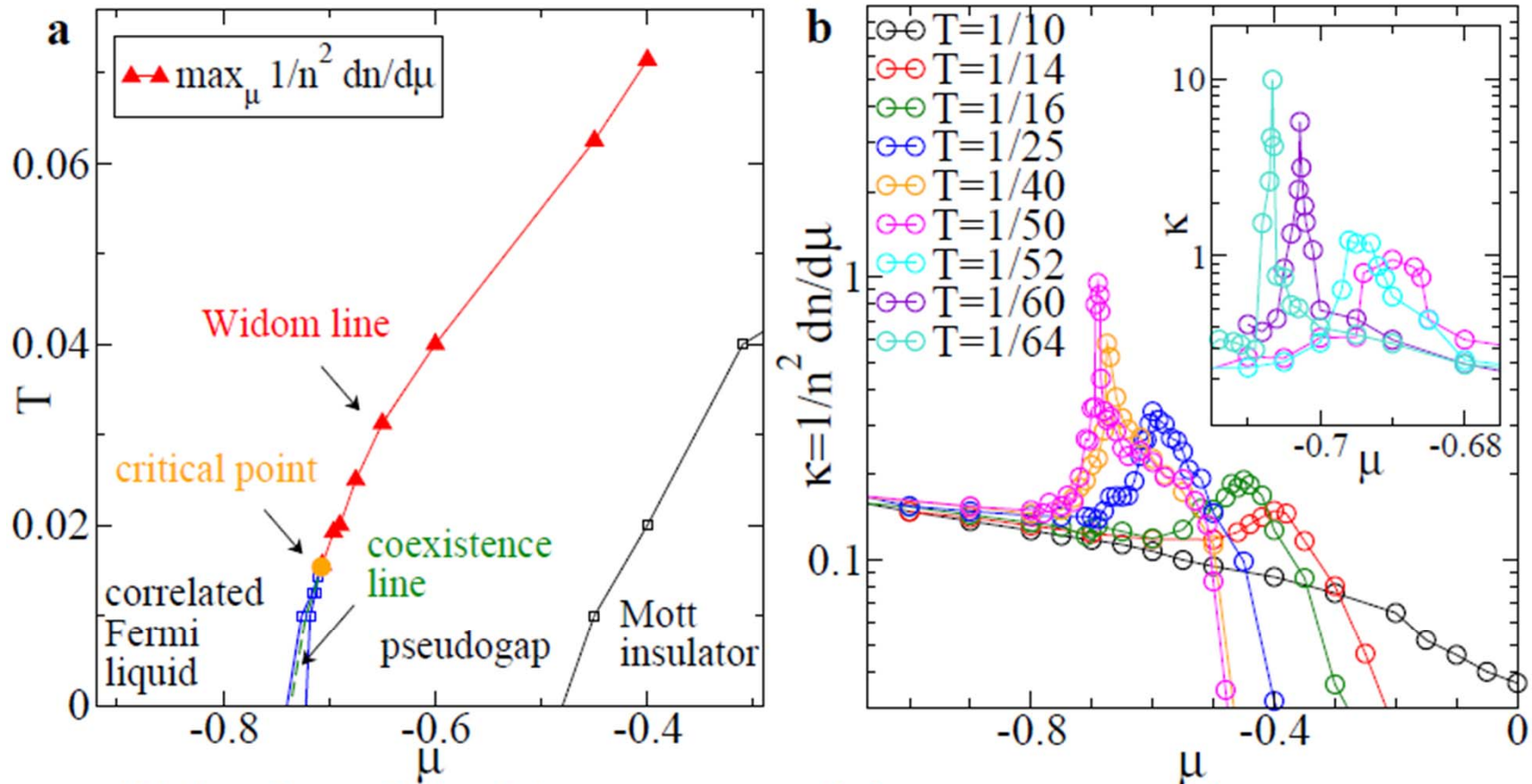
Doping dependence of critical point as a function of U



Smaller D and S



Pseudogap T^* along the Widom line



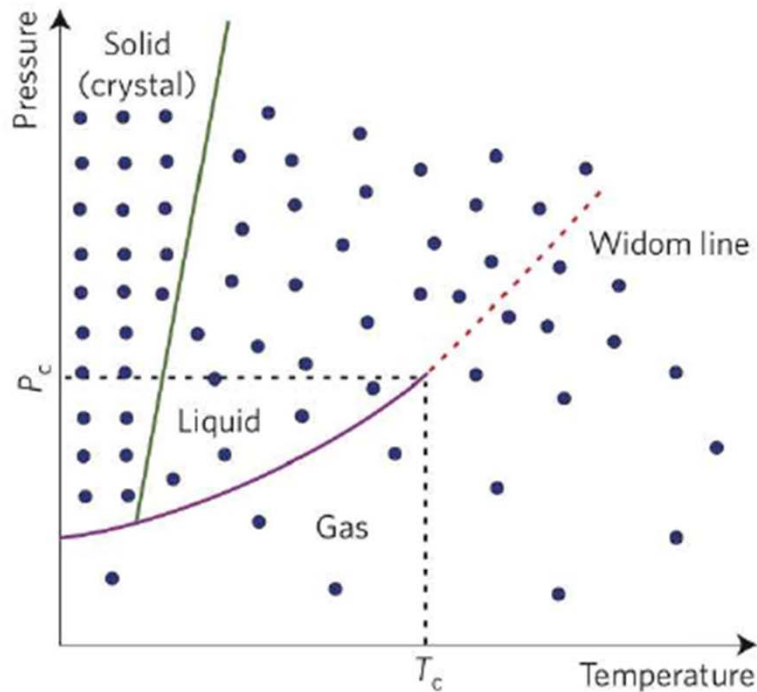
Widom line: defined from **maxima of charge compressibility**

$$\kappa = 1/n^2 (dn/d\mu) T$$

divergence of κ at the (classical) critical point!



What is the Widom line?

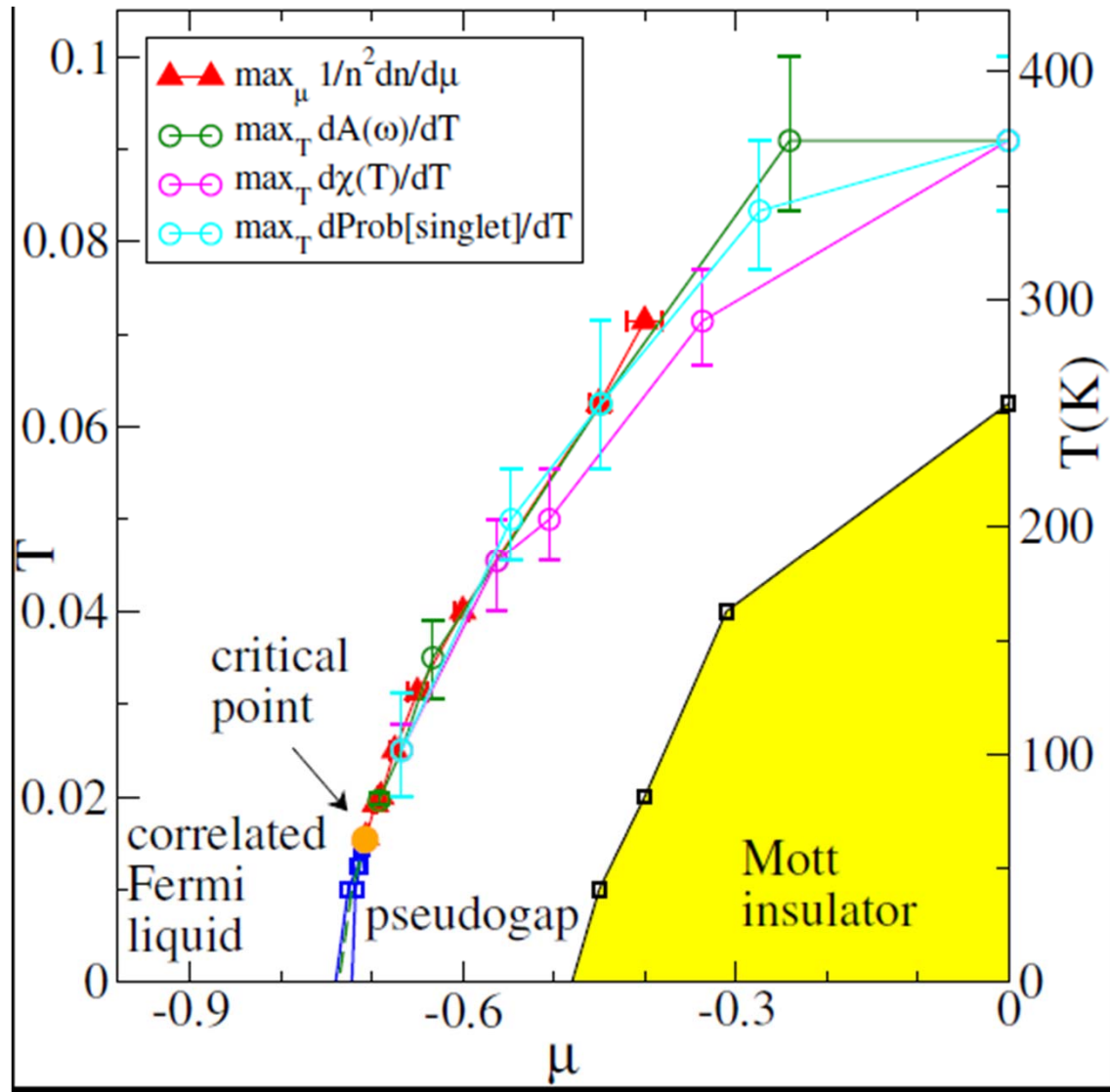


McMillan and Stanley, Nat Phys 2010

- ▶ it is the continuation of the coexistence line in the supercritical region
- ▶ line where the **maxima of different response functions** touch each other asymptotically as $T \rightarrow T_p$
- ▶ liquid-gas transition in water: max in isobaric heat capacity C_p , isothermal compressibility, isobaric heat expansion, etc
- ▶ **DYNAMIC crossover arises from crossing the Widom line!**
water: Xu et al, PNAS 2005, Simeoni et al Nat Phys 2010



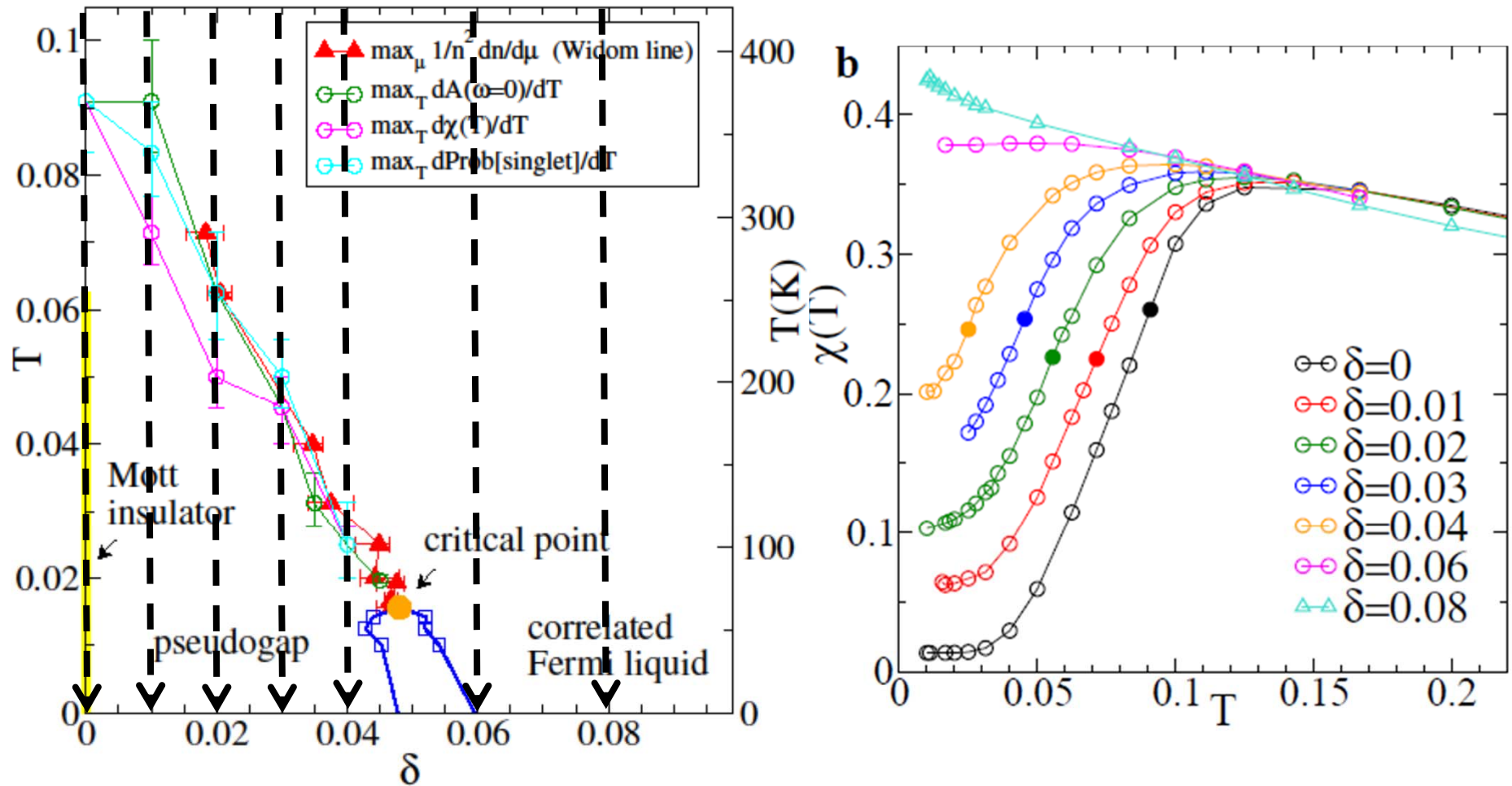
Rapid change also in dynamical quantities



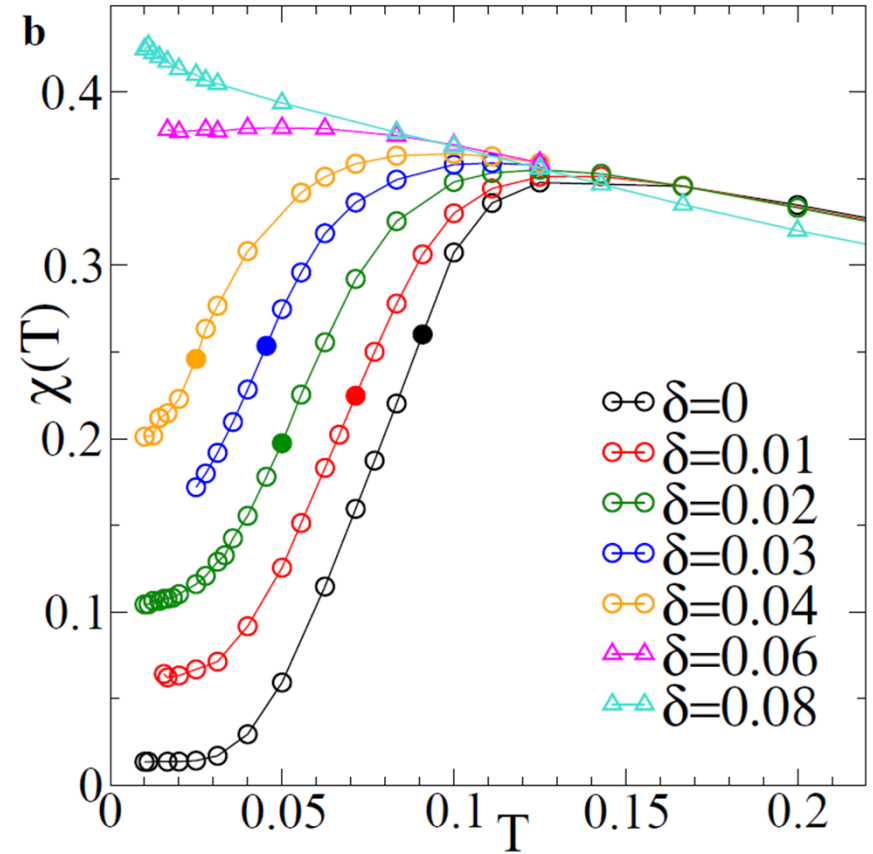
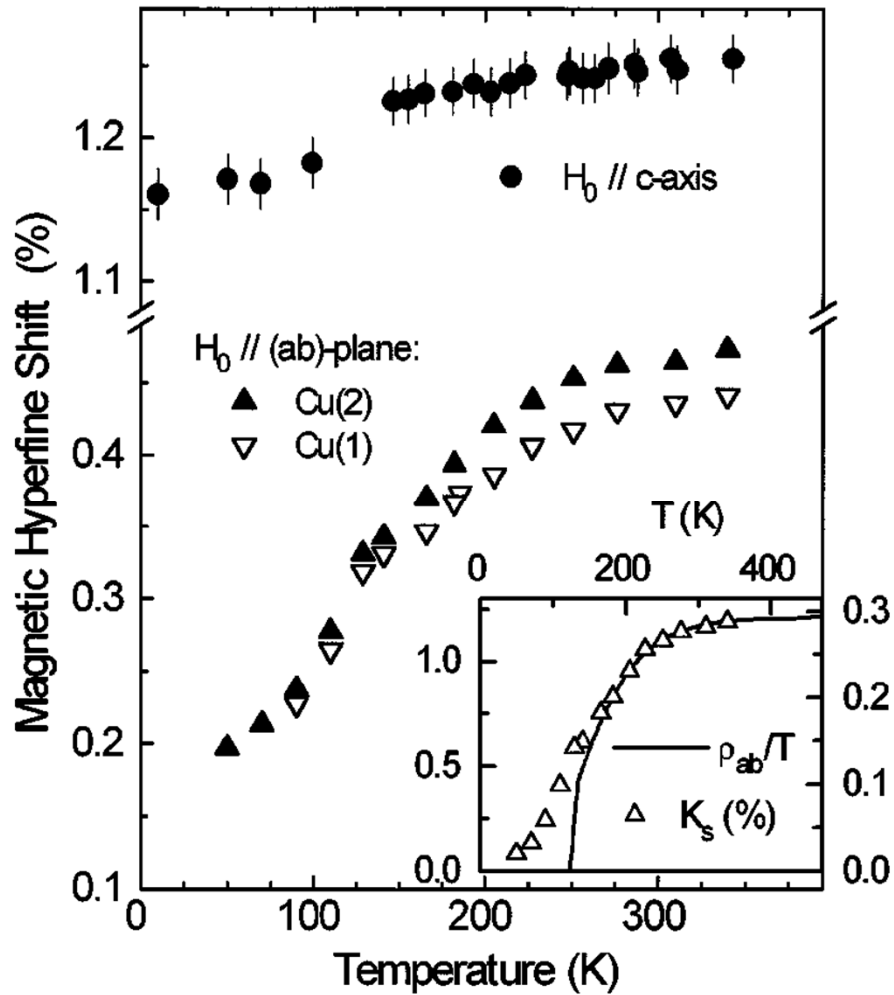
Compare a few results for cuprates



Spin susceptibility



Spin susceptibility



Underdoped Hg1223

Julien et al. PRL **76**, 4238 (1996)



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What is the minimal model?

H. Alloul arXiv:1302.3473
C.R. Académie des Sciences, (2014)

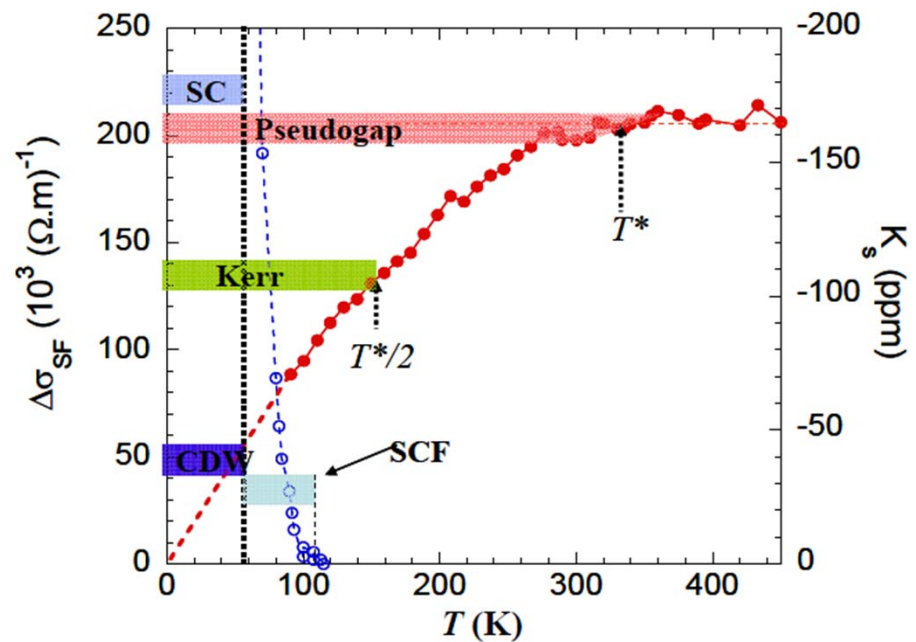
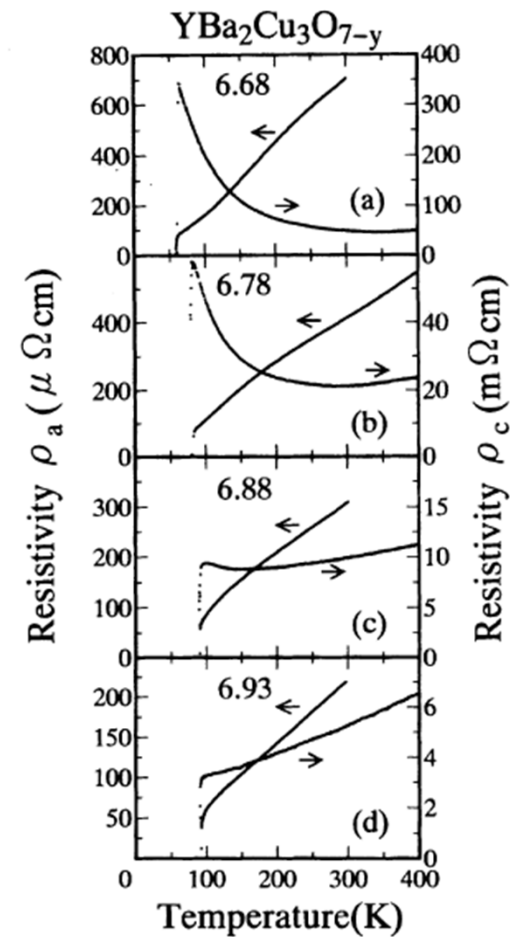
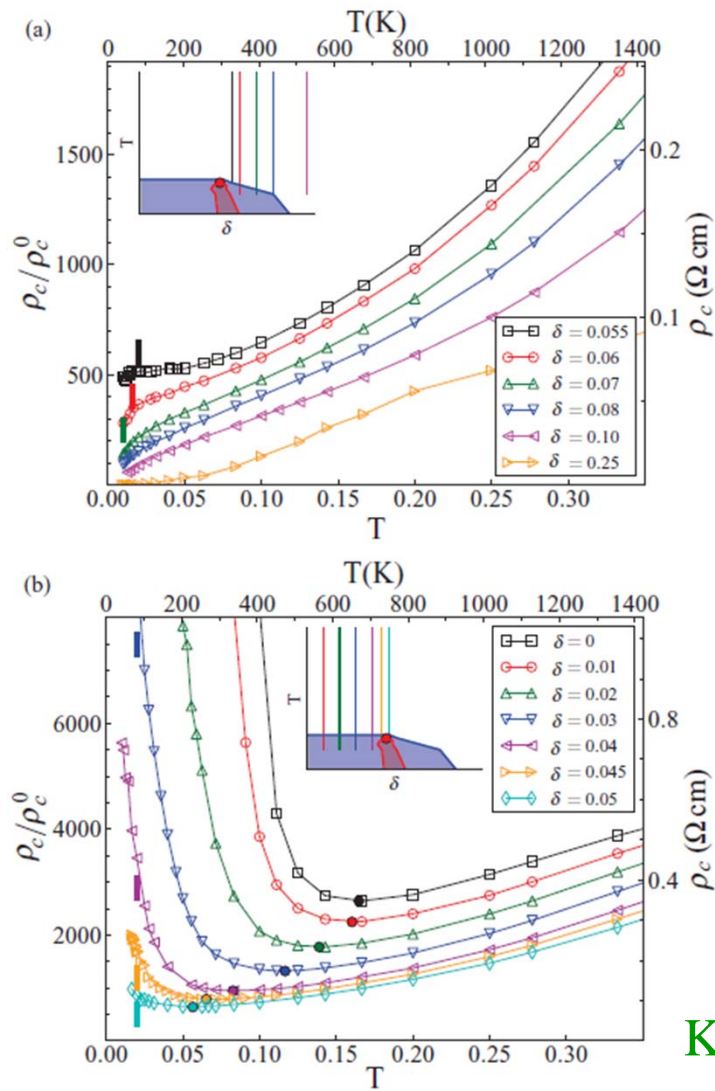


Fig 1 Spin contribution K_s to the ^{89}Y NMR Knight shift [11] for $\text{YBCO}_{6.6}$ permit to define the PG onset T^* . Here K_s is reduced by a factor two at $T \sim T^*/2$. The sharp drop of the SC fluctuation conductivity (SCF) is illustrated (left scale) [23]. We report as well the range over which a Kerr signal is detected [28], and that for which a CDW is evidenced in high fields from NMR quadrupole effects [33] and ultrasound velocity data [30]. (See text).

C-axis resistivity



K. Takenaka, K. Mizuhashi, H. Takagi, and S. Uchida,
 Phys. Rev.B 50, 6534 (1994).



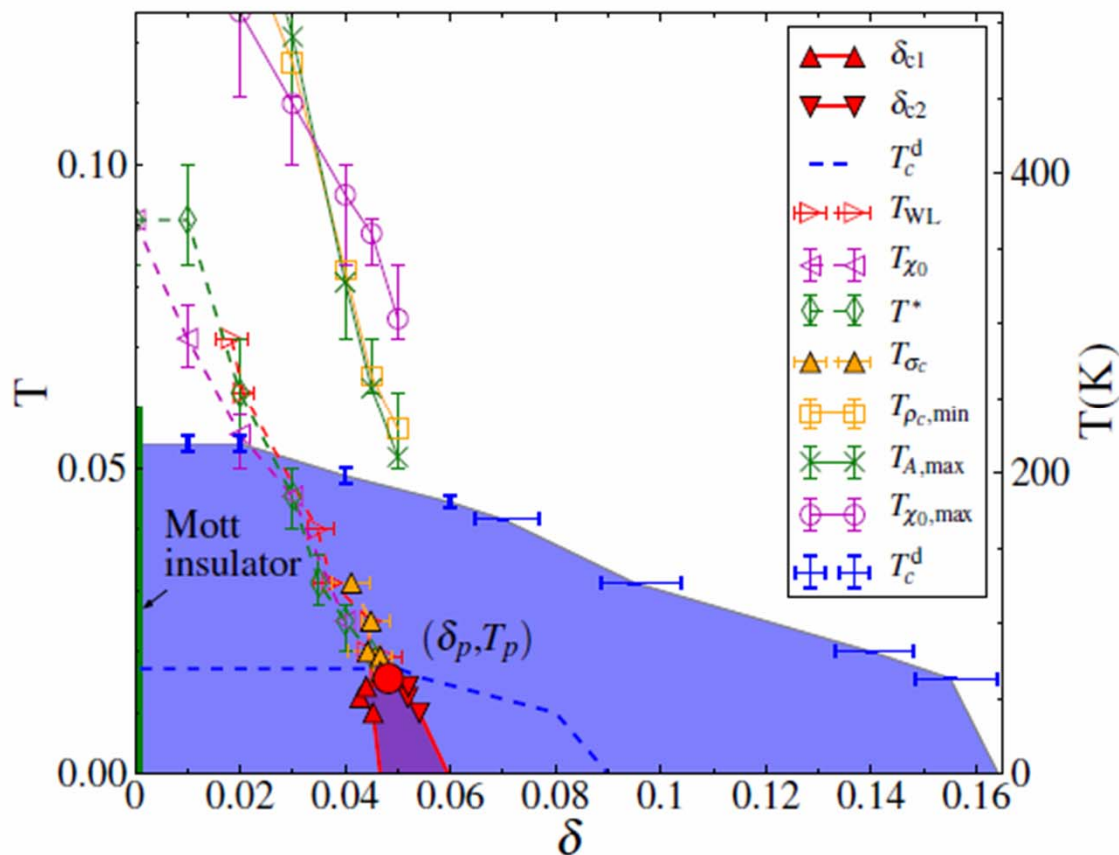


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Two crossover lines



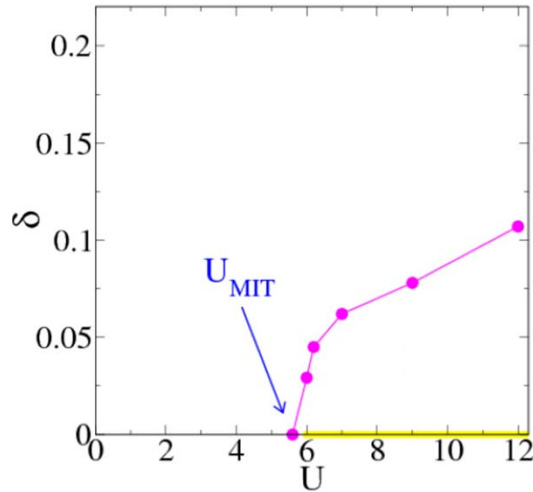
Patrick Sémon



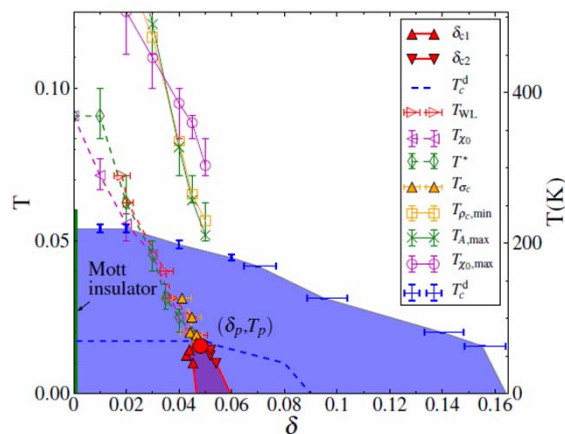
G. Sordi et al. Phys. Rev. Lett. 108, 216401/1-6 (2012)

P. Sémon, G. Sordi, A.-M.S.T., Phys. Rev. B **89**, 165113/1-6 (2014)

Summary: normal state



- Mott physics extends way beyond half-filling
- Pseudogap is a phase
- Pseudogap T^* controlled by a Widom line and its precursor
- High compressibility (stripes?)





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Patrick Sémon



Kristjan Haule

Finite T phase diagram Superconductivity

Sordi et al. PRL **108**, 216401 (2012)



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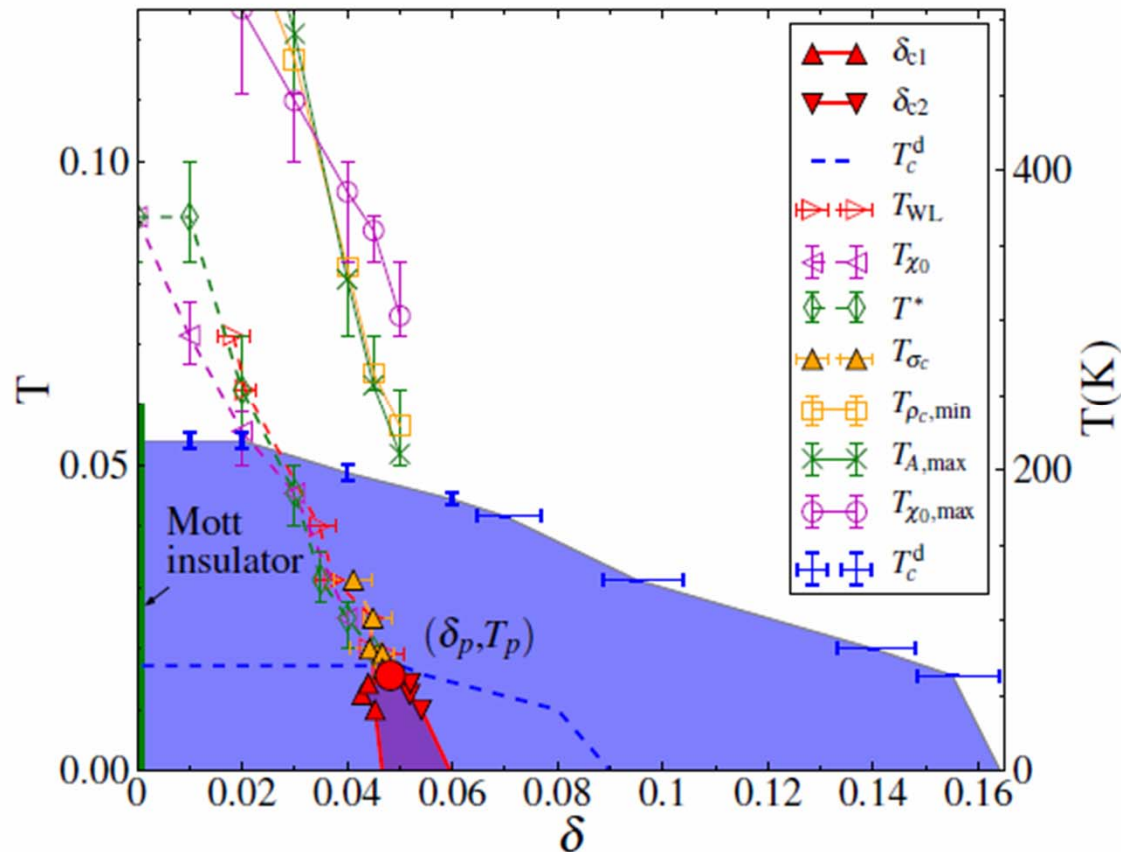


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Phase diagram for $U = 6.2 t$



Patrick Sémon



G. Sordi et al. Phys. Rev. Lett. 108, 216401/1-6 (2012)

P. Sémon, G. Sordi, A.-M.S.T., Phys. Rev. B **89**, 165113/1-6 (2014)

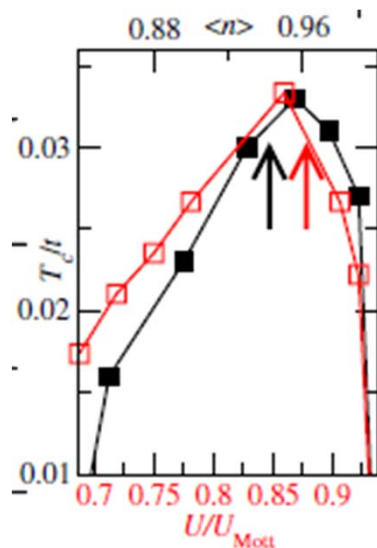
Actual T_c in underdoped

- Quantum and classical phase fluctuations
 - V. J. Emery and S. A. Kivelson, Phys. Rev. Lett. **74**, 3253 (1995).
 - V. J. Emery and S. A. Kivelson, Nature **374**, 474 (1995).
 - D. Podolsky, S. Raghu, and A. Vishwanath, Phys. Rev. Lett. **99**, 117004 (2007).
 - Z. Tesanovic, Nat Phys **4**, 408 (2008).
- Magnitude fluctuations
 - I. Ussishkin, S. L. Sondhi, and D. A. Huse, Phys. Rev. Lett. **89**, 287001 (2002).
- Competing order
 - E. Fradkin, S. A. Kivelson, M. J. Lawler, J. P. Eisenstein, and A. P. Mackenzie, Annual Review of Condensed Matter Physics **1**, 153 (2010).
- Disorder
 - F. Rullier-Albenque, H. Alloul, F. Balakirev, and C. Proust, EPL (Europhysics Letters) **81**, 37008 (2008).
 - H. Alloul, J. Bobro, M. Gabay, and P. J. Hirschfeld, Rev. Mod. Phys. **81**, 45 (2009).

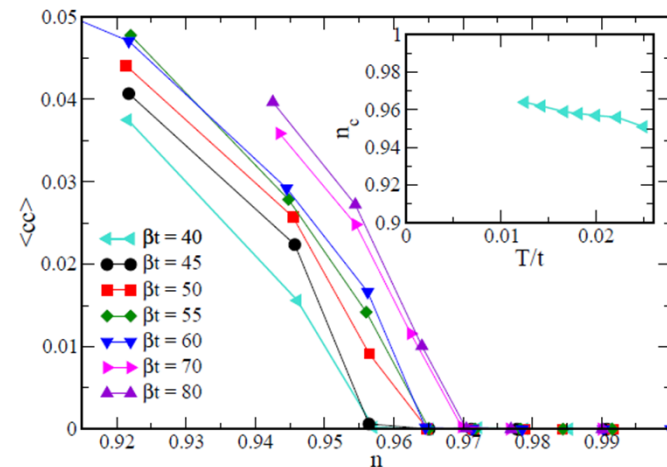


Larger clusters

- In 2×2 T_c vanishes extremely close to half-filling. In larger cluster, earlier.
- Local pairs in underdoped (2×2)



8 site DCA, $U=6t$

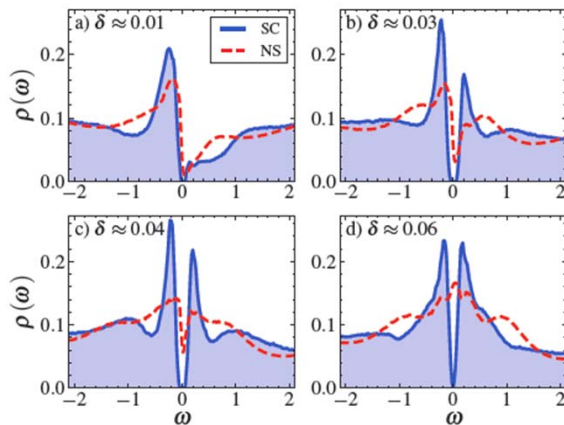
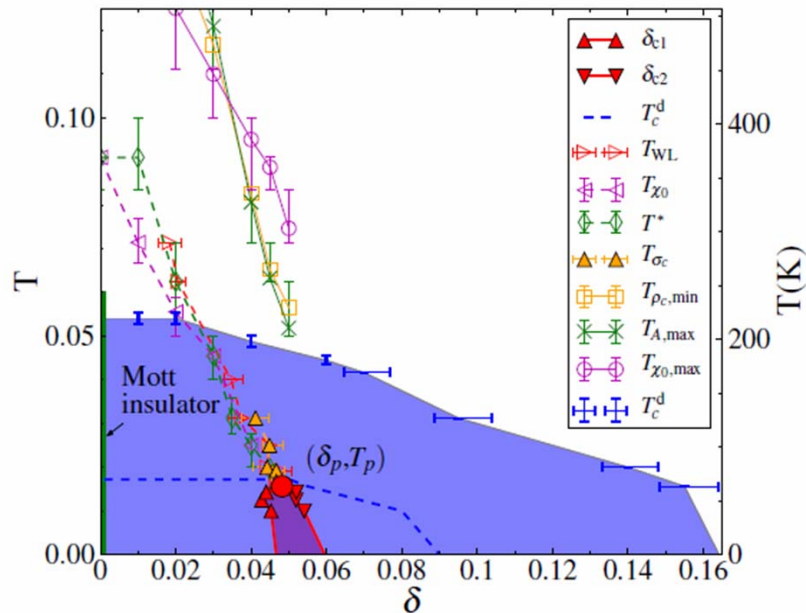


8 site DCA, $U=6.5t$

Gull Parcollet Millis,
PRL **110**, 216405 (2013)



Summary

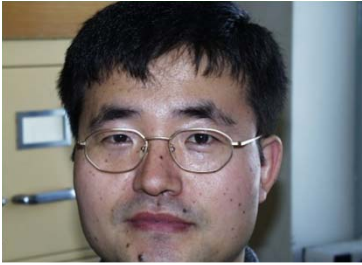


- Below the dome, not QCP (but Mott)
- Maximum at Widom line
- T^* different from T_c^d
- First-order transition destroyed but traces in the dynamics
- Actual T_c in underdoped
 - Competing order
 - Long wavelength fluctuations (see O.P.)
 - Disorder

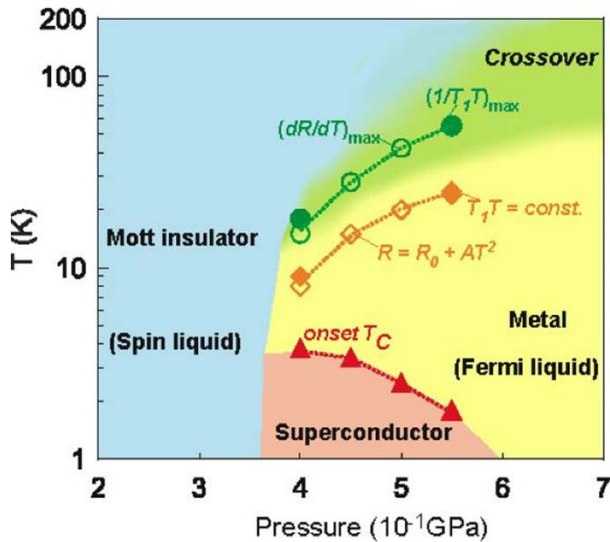


Bandwidth control and doping control of the Mott transition in organics



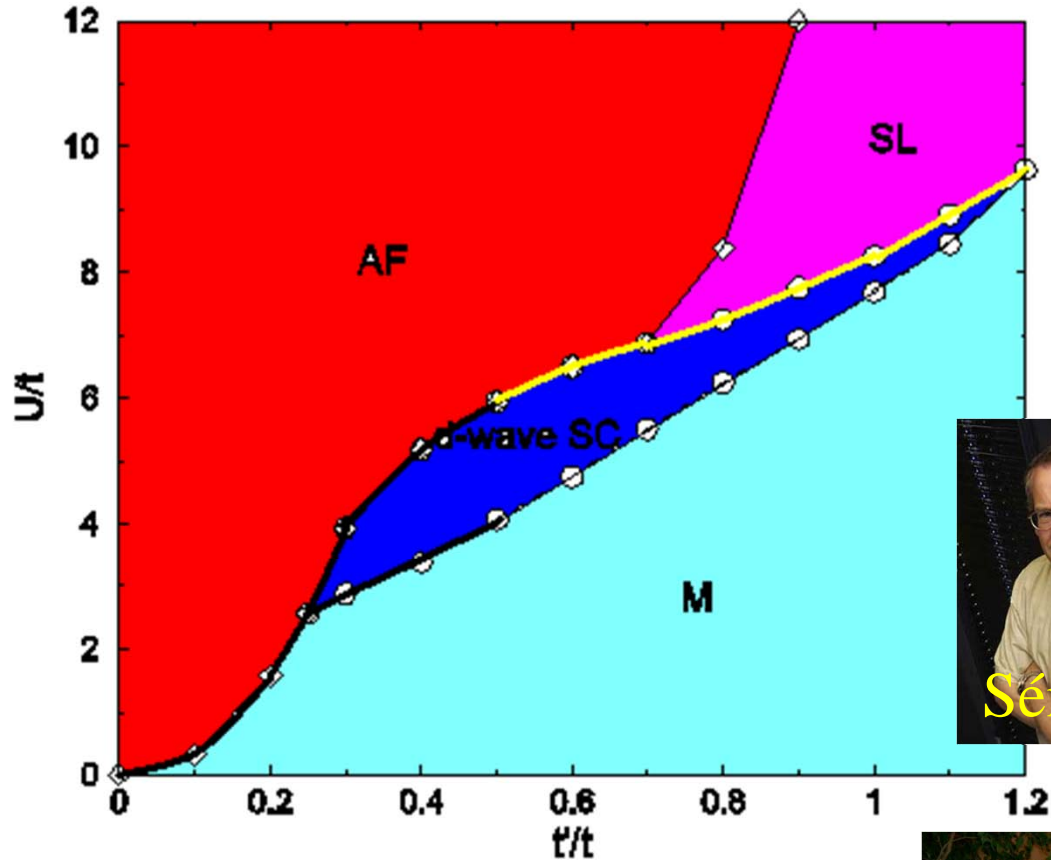


Theoretical phase diagram BEDT



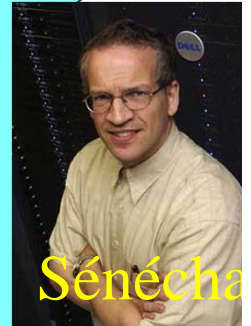
Y. Kurisaki, et al.

Phys. Rev. Lett. **95**, 177001(2005) Y. Shimizu, et al. Phys. Rev. Lett. **91**, (2003)

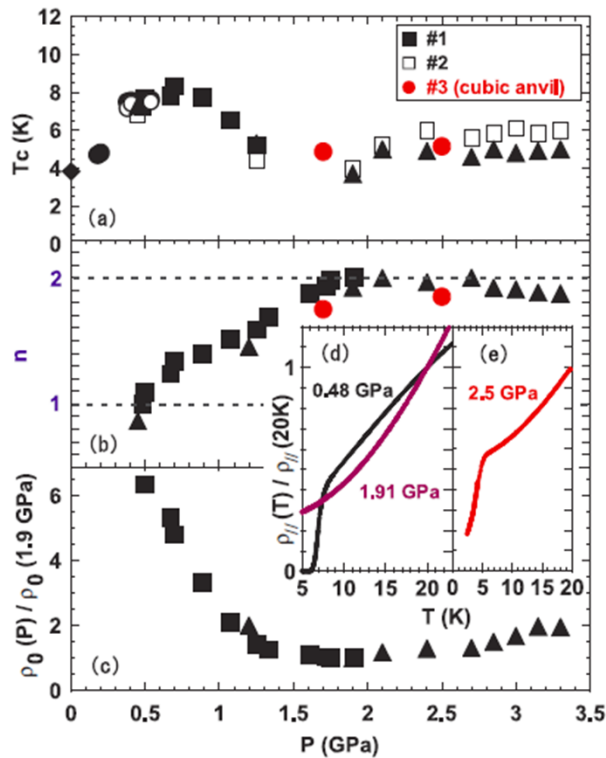


Kyung, A.-M.S.T. PRL 97, 046402 (2006)

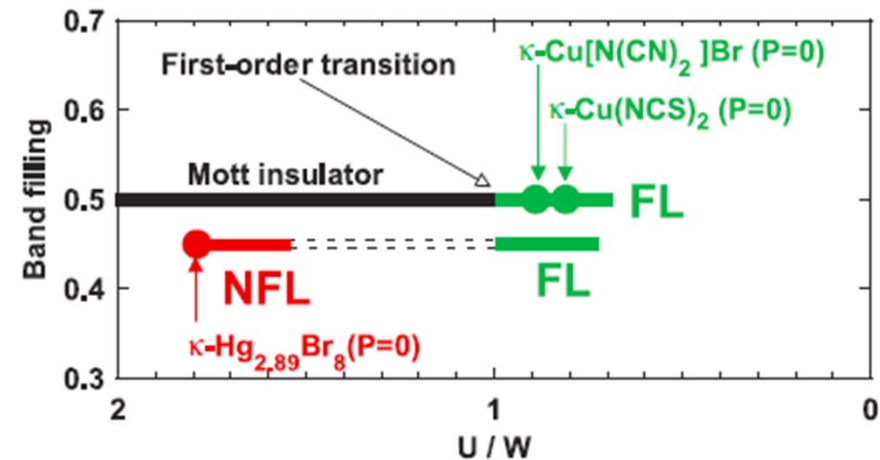
Sénéchal, Sahebsara, Phys. Rev. Lett. **97**, 257004



A doped BEDT organic



	W (eV)	U (eV)	U/W	BF	T_c (K)
$\kappa\text{-Cu(NCS)}_2$ ^{a)}	0.57	0.46	0.81	0.50	10.4
$\kappa\text{-Cu[N(CN)}_2\text{]Br}$ ^{a)}	0.55	0.49	0.89	0.50	11.8
$\kappa\text{-Hg}_{2.89}\text{Br}_8$ ^{b)}	0.26	0.465	1.79	0.45	4.3

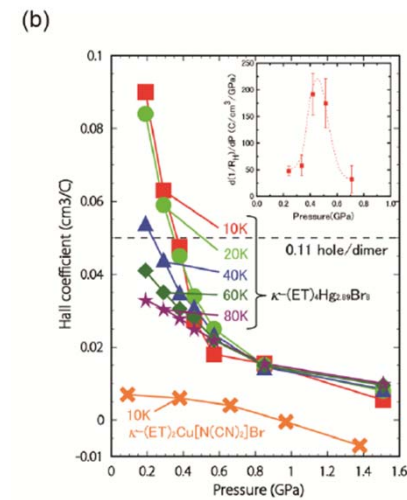
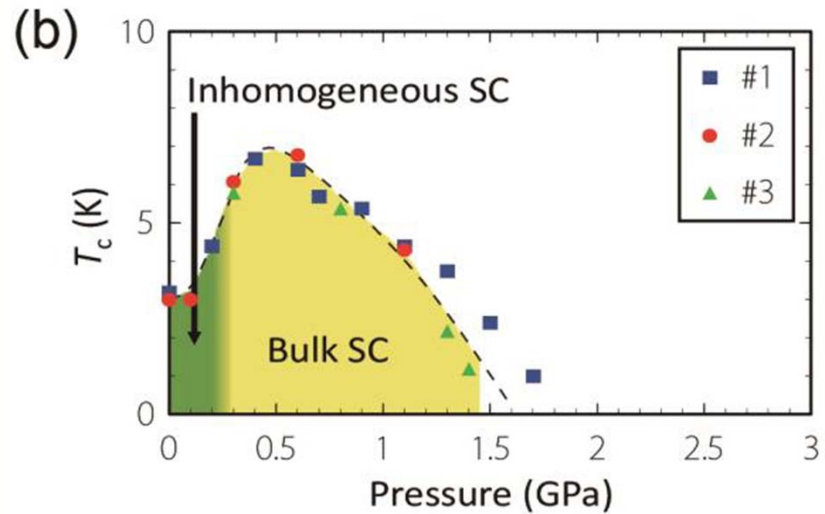
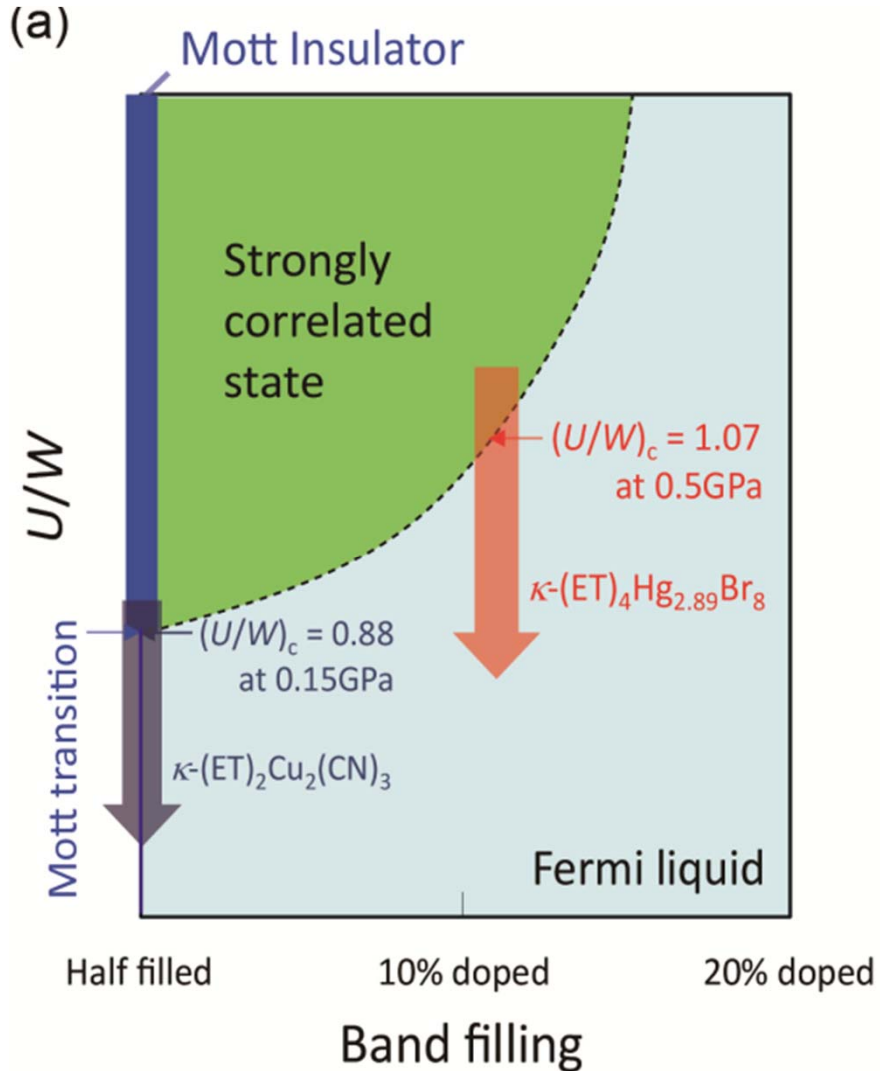


Taniguchi et al. J. Phys. Soc. Japan, **76**, 113709 (2007)

R. N. Lyubovskaya et al. JETP Lett. **45**, 530 (1987)



Doped BEDT

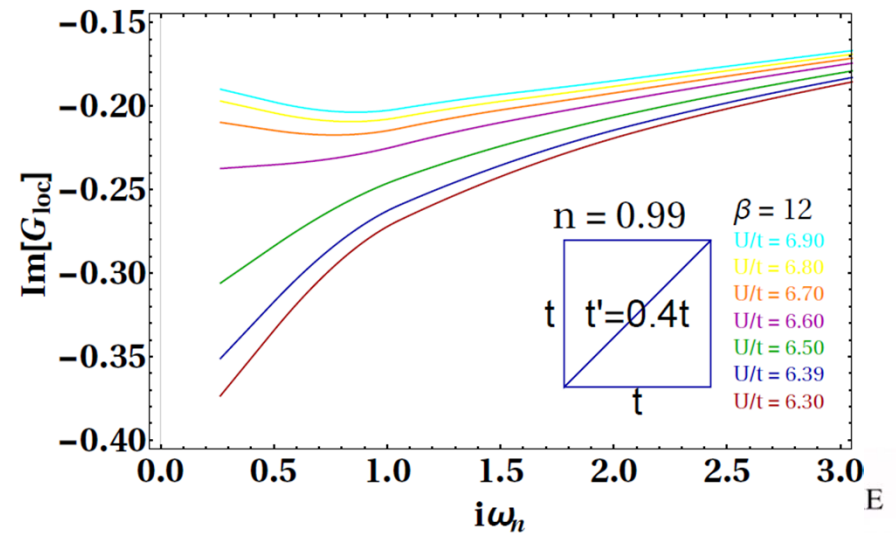
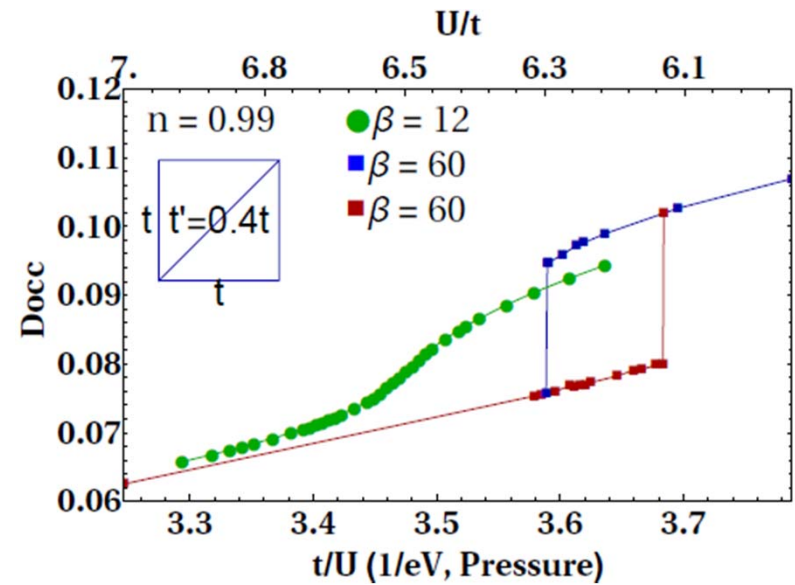
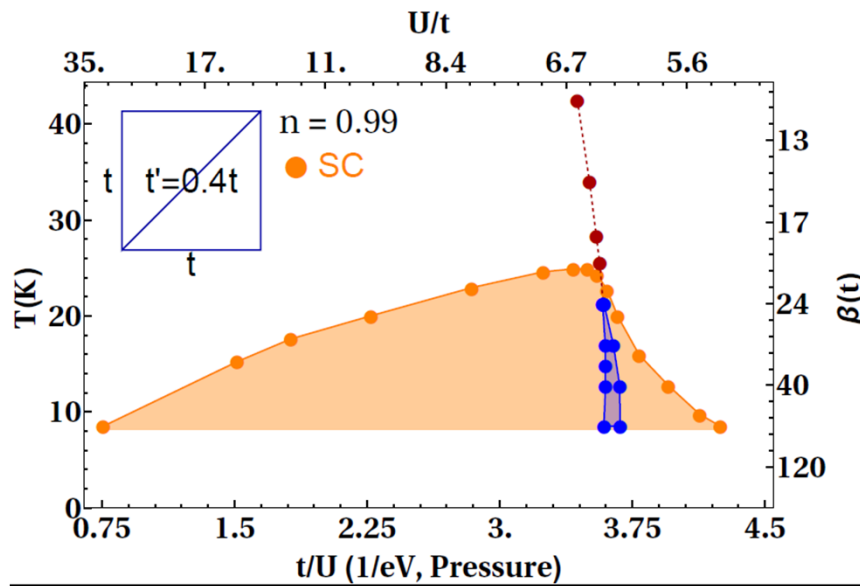


H. Oike, K. Miyagawa, H. Taniguchi, K. Kanoda PRL **114**, 067002 (2015)



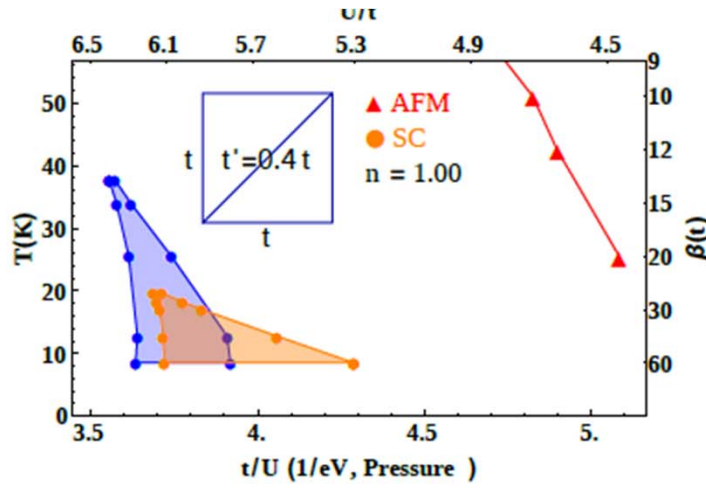
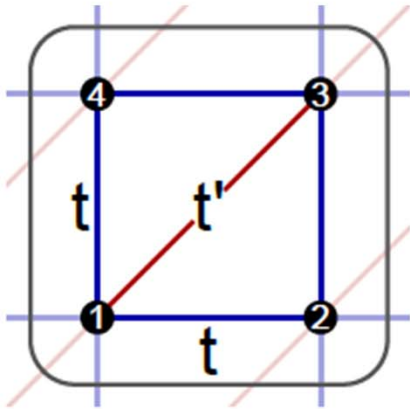
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Widom line in organics

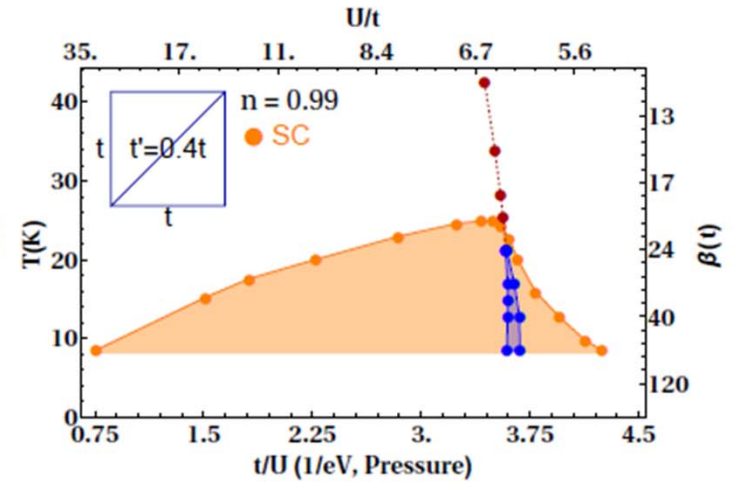


Charles-David Hébert, Patrick Sémon, AMT

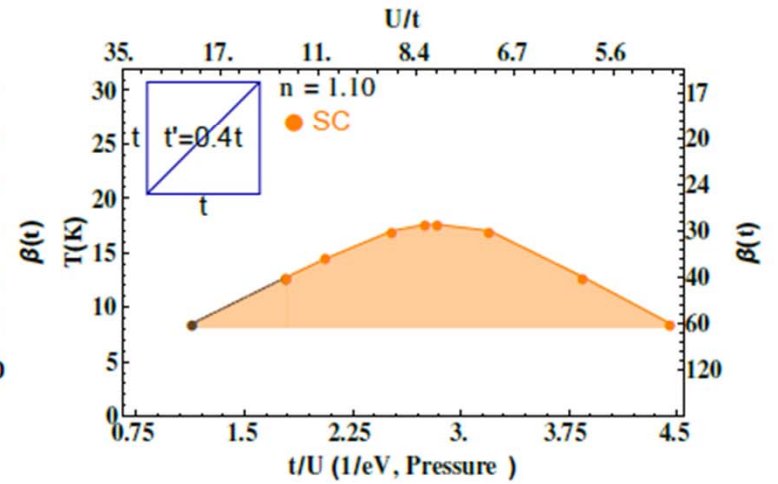
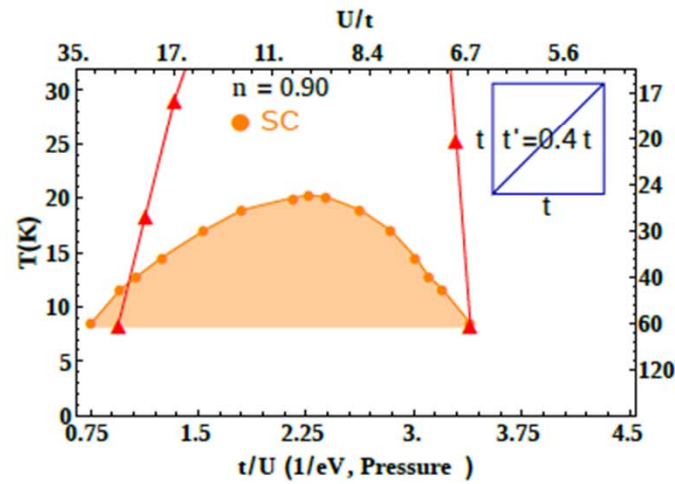
$$t' = 0.4t$$



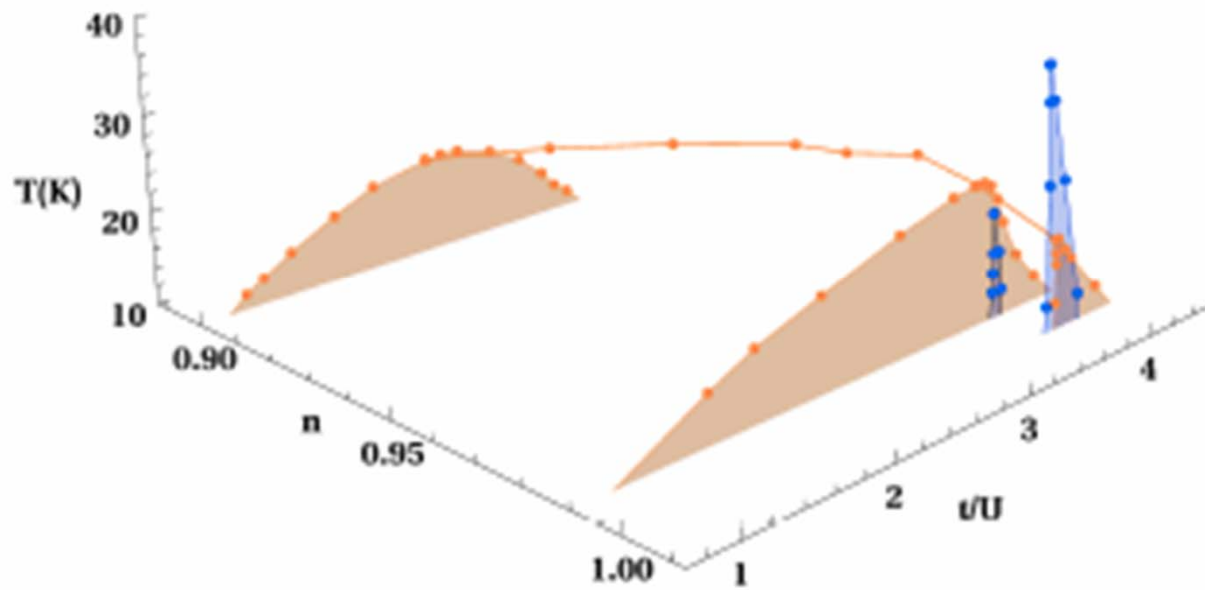
(a)



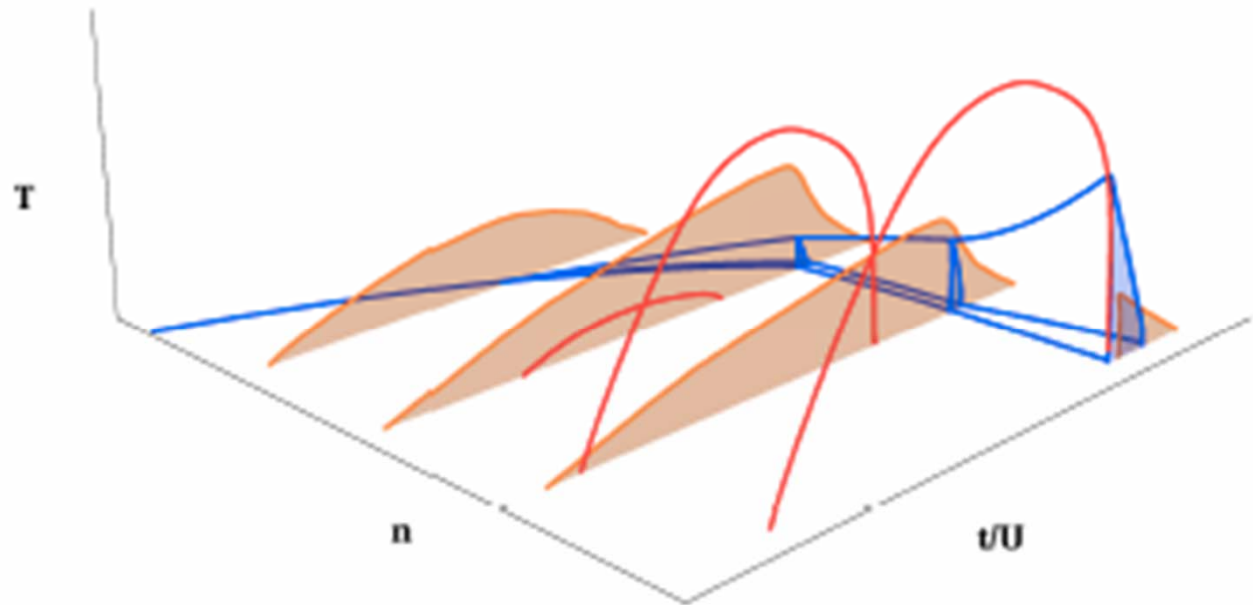
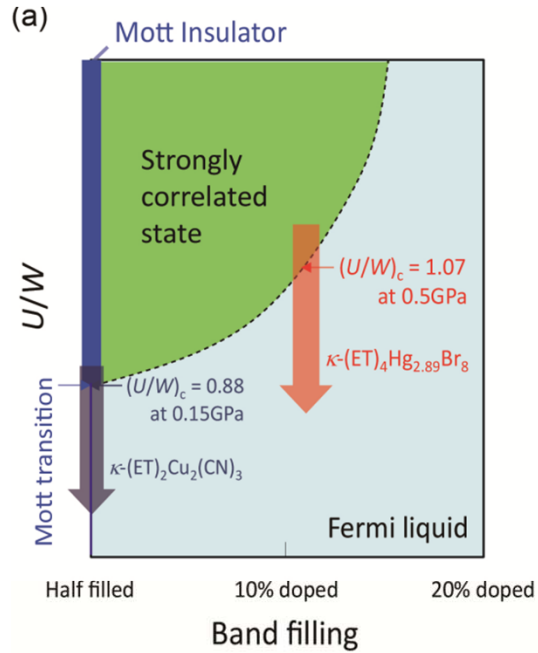
(b)



$t' = 0.4t$ overview



Generic case



Summary : organics

- Agreement with experiment
 - SC: larger T_c and broader P range if doped
 - Larger frustration: Decrease T_N and T_c
 - Normal state metal to pseudogap crossover
- Predictions
 - First order transition at low T in normal state (or remnants in SC state)
- Physics
 - SC dome without a QCP. Follows first-order.
 - SC from short range J .
 - T_c decreases at Widom line



Some Algorithmic details: 3 improvements



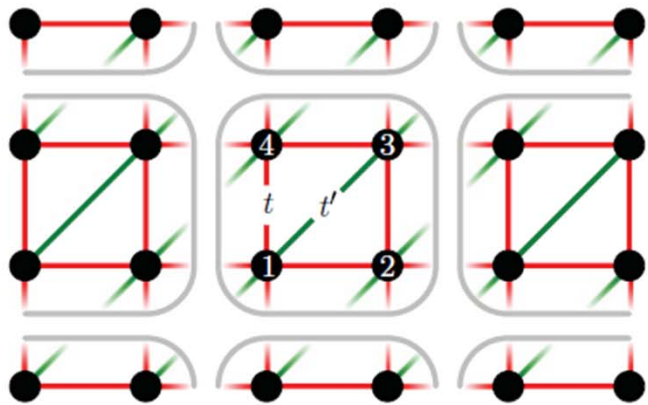
Continuous-time QMC : CT-HYB

$$H_{\text{imp}} = H_{\text{loc}}(d_i^\dagger, d_i) + \sum_{i\mu} (V_{\mu i} a_\mu^\dagger d_i + V_{\mu i}^* d_i^\dagger a_\mu) + \sum_{\mu} \epsilon_{\mu} a_{\mu}^{\dagger} a_{\mu},$$

$$\begin{aligned} Z &= \text{Tr} T_{\tau} e^{-\beta H_0} e^{-\int_0^{\beta} d\tau (H_{\text{hyb}}(\tau) + H_{\text{hyb}}^{\dagger}(\tau))} \\ &= \sum_{k \geq 0} \frac{1}{k!^2} \int_0^{\beta} d\tau_1 \cdots d\tau_k \int_0^{\beta} d\tau'_1 \cdots d\tau'_k \text{Tr} T_{\tau} e^{-\beta H_0} \\ &\quad \times H_{\text{hyb}}(\tau_1) H_{\text{hyb}}^{\dagger}(\tau'_1) \cdots H_{\text{hyb}}(\tau_k) H_{\text{hyb}}^{\dagger}(\tau'_k). \\ &= \sum_{k \geq 0} \sum_{i_1 \cdots i_k} \sum_{i'_1 \cdots i'_k} \frac{1}{k!^2} \int_0^{\beta} d\tau_1 \cdots d\tau_k \int_0^{\beta} d\tau'_1 \cdots d\tau'_k \\ &\quad \times \text{Tr} T_{\tau} e^{-\beta H_{\text{loc}}} d_{i_1}(\tau_1) d_{i'_1}^{\dagger}(\tau'_1) \cdots d_{i_k}(\tau_k) d_{i'_k}^{\dagger}(\tau'_k) \\ &\quad \times Z_{\text{bath}} \langle \hat{V}_{i_1}^{\dagger}(\tau_1) \hat{V}_{i'_1}(\tau'_1) \cdots \hat{V}_{i_k}^{\dagger}(\tau_k) \hat{V}_{i'_k}(\tau'_k) \rangle, \quad \hat{V}_i = \sum_{\mu} V_{\mu i}^* a_{\mu} \end{aligned}$$

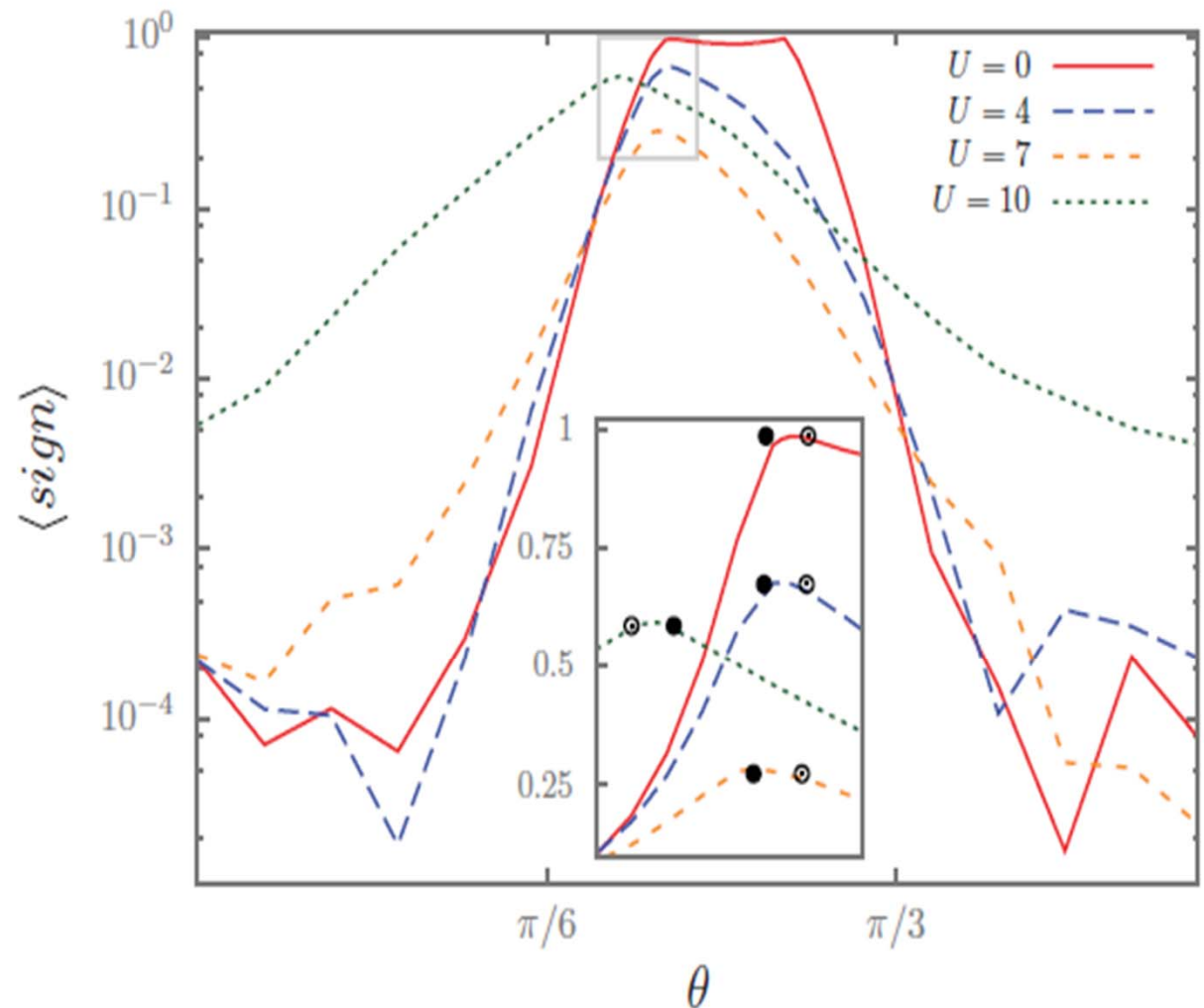
Reducing the sign problem

$$\cos \theta c'_{A_1\sigma} - \sin \theta c_{A_1\sigma}, \quad \sin \theta c'_{A_1\sigma} + \cos \theta c_{A_1\sigma}$$



$$t'/t = 0.8$$

$$C_{2v} \\ 2A_1, B_1, B_2$$



Ergodicity of the hybridization expansion with two operator updates and broken symmetry

$$H_{\text{imp}} = H_{\text{loc}}(d_i^\dagger, d_i) + \sum_{i\mu} (V_{\mu i} a_\mu^\dagger d_i + V_{\mu i}^* d_i^\dagger a_\mu) + \sum_{\mu} \epsilon_{\mu} a_{\mu}^{\dagger} a_{\mu},$$



Patrick Sémon

$$Z = \text{Tr} \mathcal{T}_{\tau} e^{-\beta H_0} e^{-\int_0^{\beta} d\tau (H_{\text{hyb}}(\tau) + H_{\text{hyb}}^{\dagger}(\tau))}$$

$$= \sum_{k \geq 0} \frac{1}{k!^2} \int_0^{\beta} d\tau_1 \cdots d\tau_k \int_0^{\beta} d\tau'_1 \cdots d\tau'_k \text{Tr} \mathcal{T}_{\tau} e^{-\beta H_0}$$

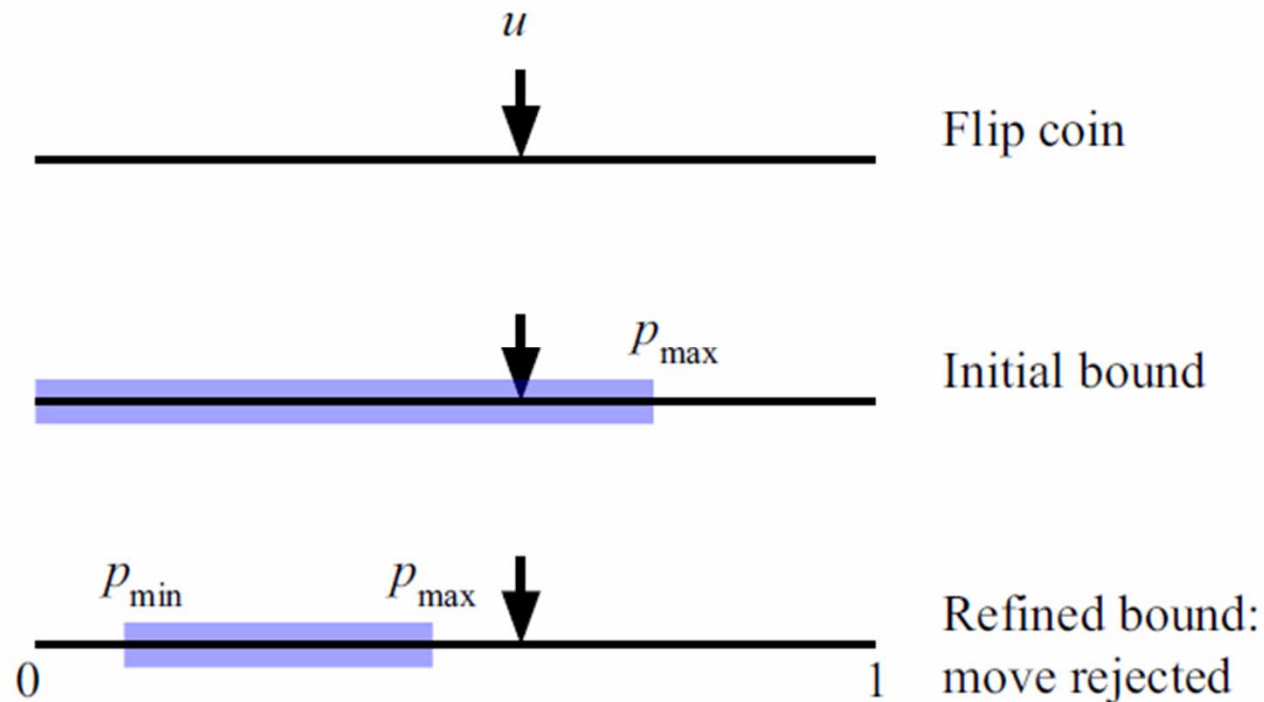
$$\times H_{\text{hyb}}(\tau_1) H_{\text{hyb}}^{\dagger}(\tau'_1) \cdots H_{\text{hyb}}(\tau_k) H_{\text{hyb}}^{\dagger}(\tau'_k).$$

$$\text{Tr} [d_{\uparrow(0,\pi)} d_{\downarrow(0,\pi)} d_{\downarrow(\pi,0)}^{\dagger} d_{\uparrow(\pi,0)}^{\dagger}]$$

$$\times \Delta_{a_{\uparrow(0,\pi),\downarrow(0,\pi)}} \Delta_{a_{\uparrow(\pi,0),\downarrow(\pi,0)}}$$

Lazy Skip-List : 1 Lazy

Fast rejection algorithm : the lazy part



P. Sémon, Chuck-Hou Yee, K. Haule, A.-M.S. Tremblay, Phys. Rev. B **90**, 075149 (2014)



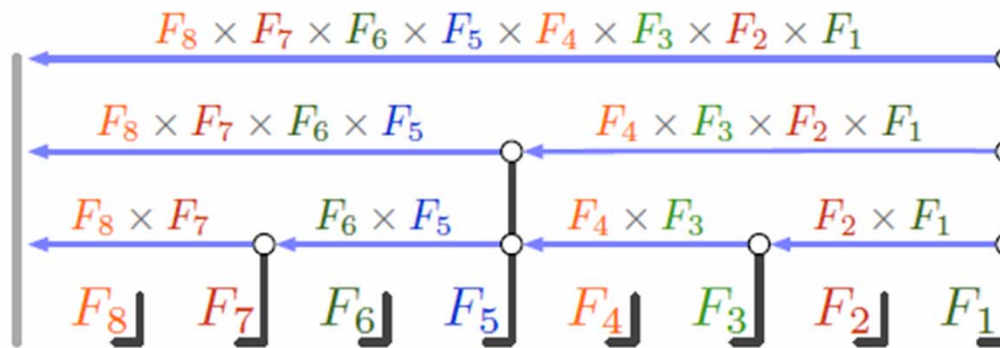
MC weights in CT-HYB some notation

$$w\{(i_1, \tau_1) \cdots (i'_k, \tau'_k)\} = \text{Det } \Delta \text{Tr}_{\text{loc}} \left[T_\tau e^{-\beta H_{\text{loc}}} \right. \\ \left. \times d_{i_k}(\tau_k) d_{i'_k}^\dagger(\tau'_k) \cdots d_{i_1}(\tau_1) d_{i'_1}^\dagger(\tau'_1) \right]$$

$$\text{Tr}_{\text{loc}} P_{\beta-\tau_k} F_{i_k} P_{\tau_k-\tau'_k} F_{i'_k}^\dagger \cdots F_{i_1} P_{\tau_1-\tau'_1} F_{i'_1}^\dagger P_{\tau'_1}$$

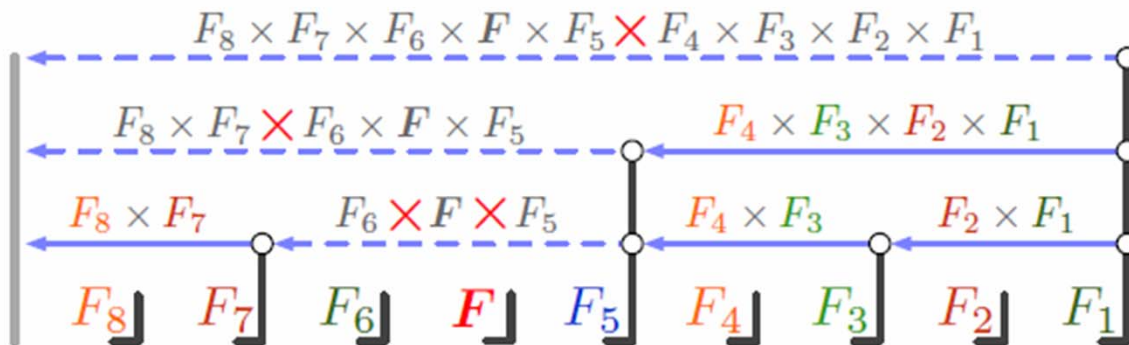
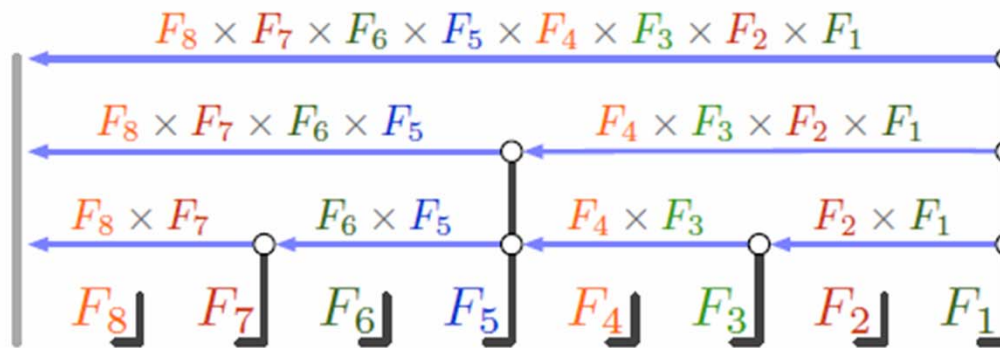


Lazy Skip List : Skip List



Tree structure : E. Gull, ETH thesis

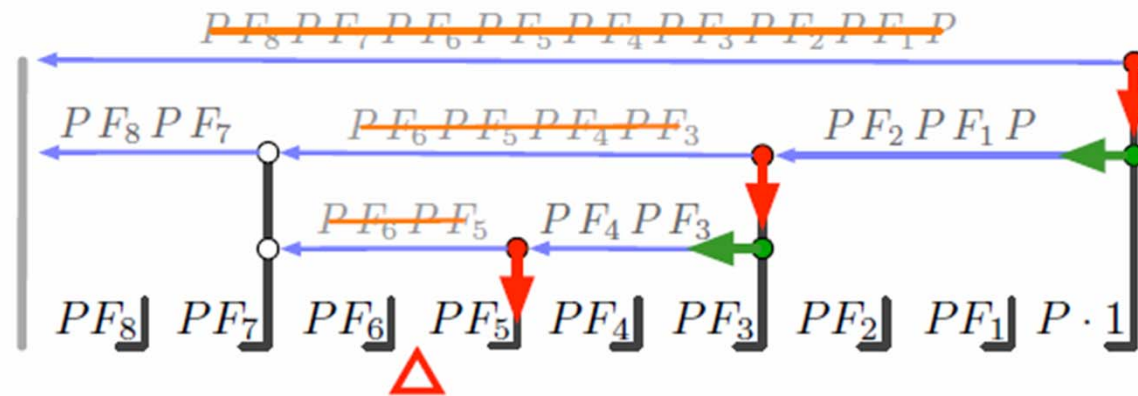
Lazy Skip List : Skip List



Tree structure : E. Gull, ETH thesis

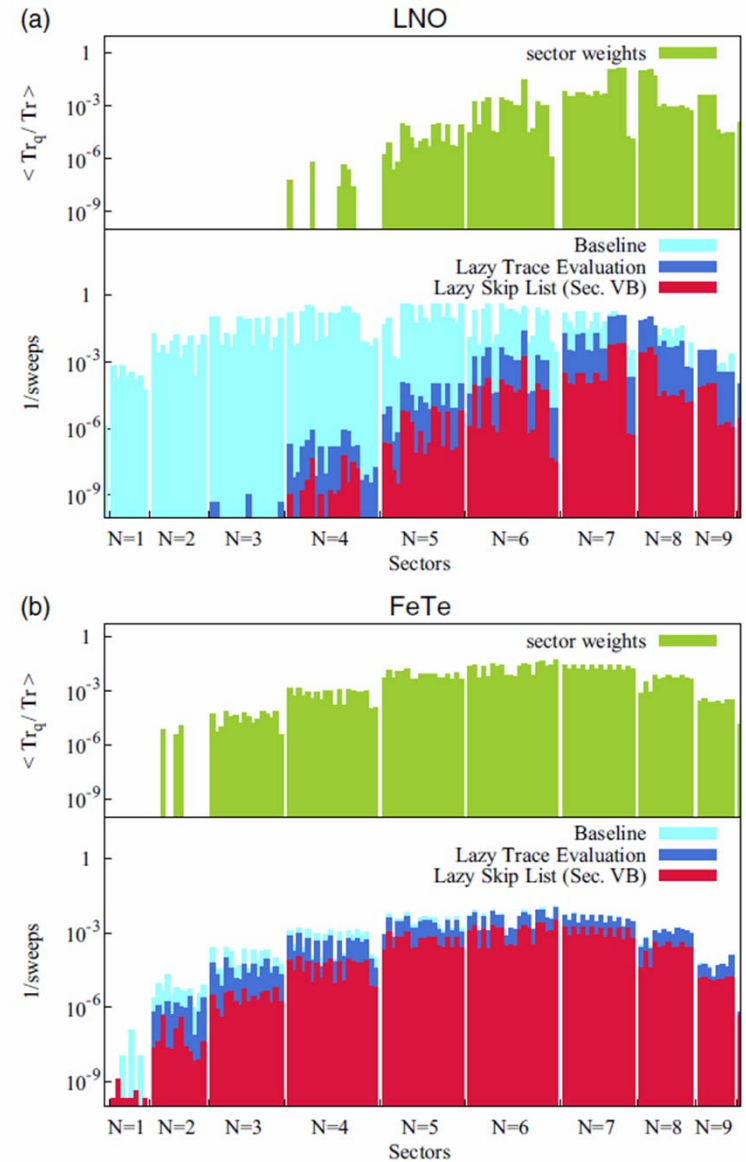
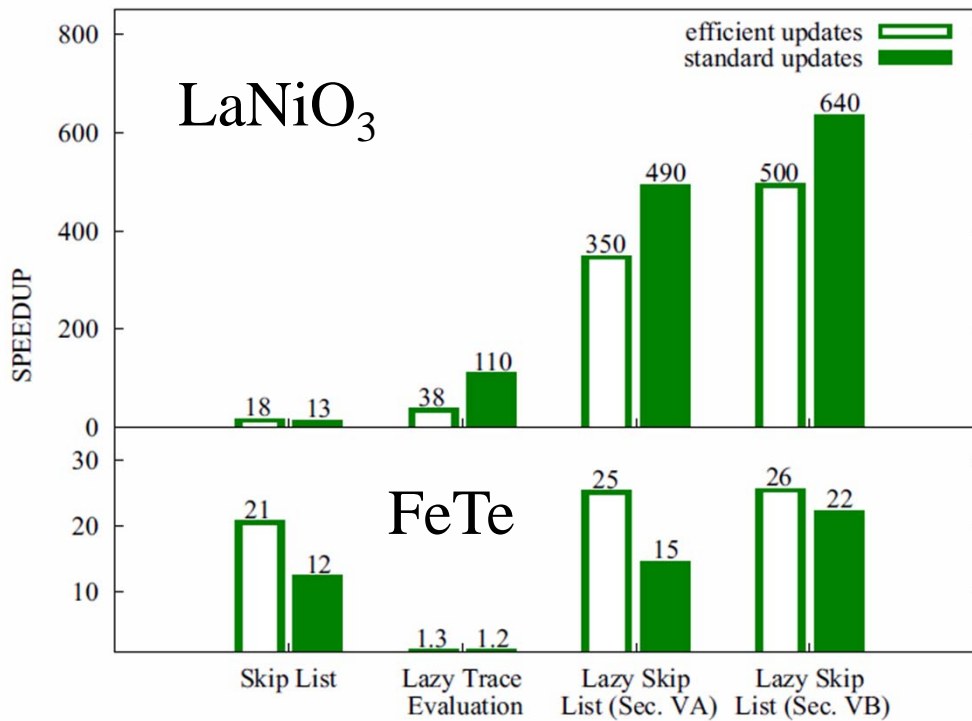


Some more details

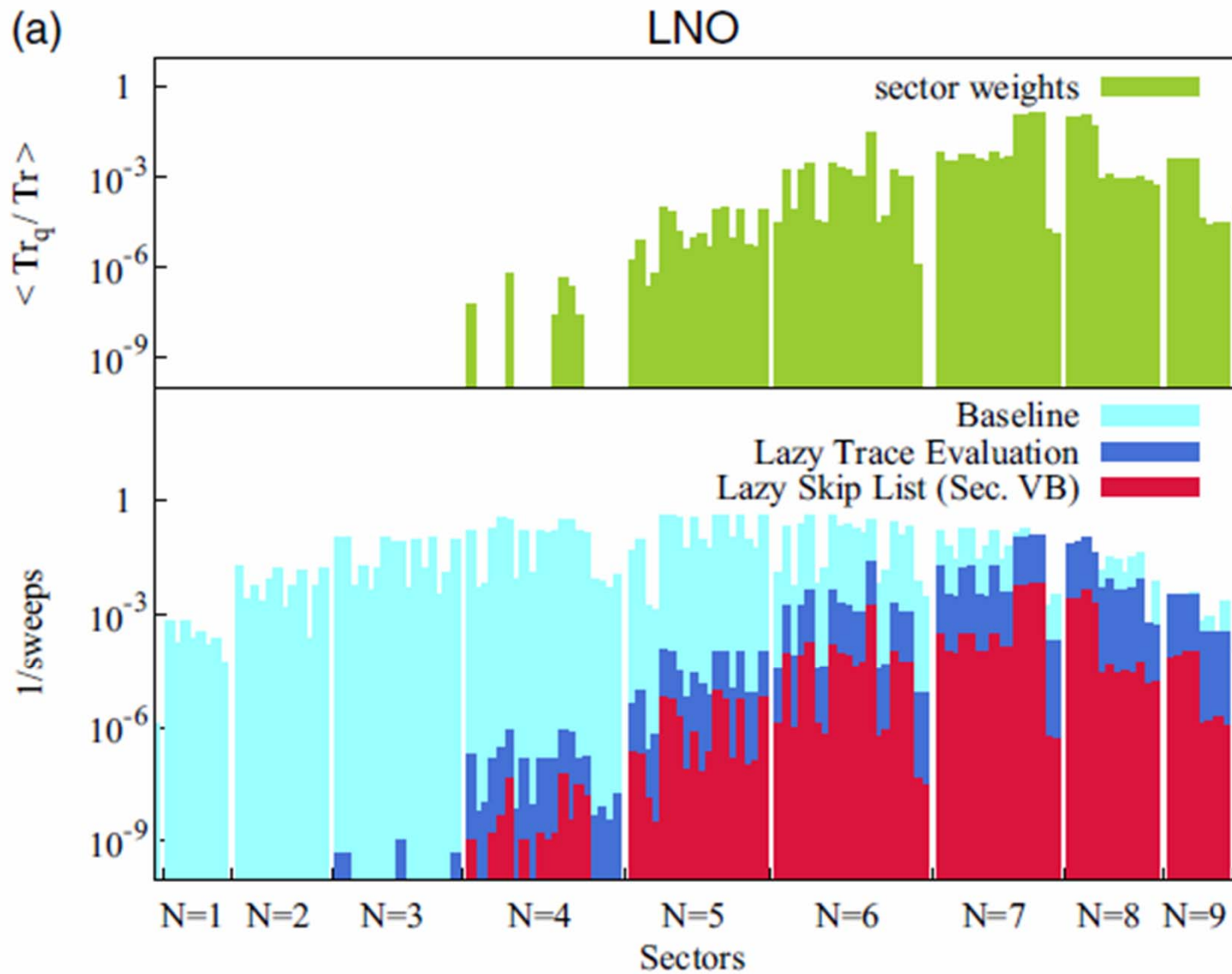


Subproducts stored in blue arrows are emptied
if tail coincides with red arrow

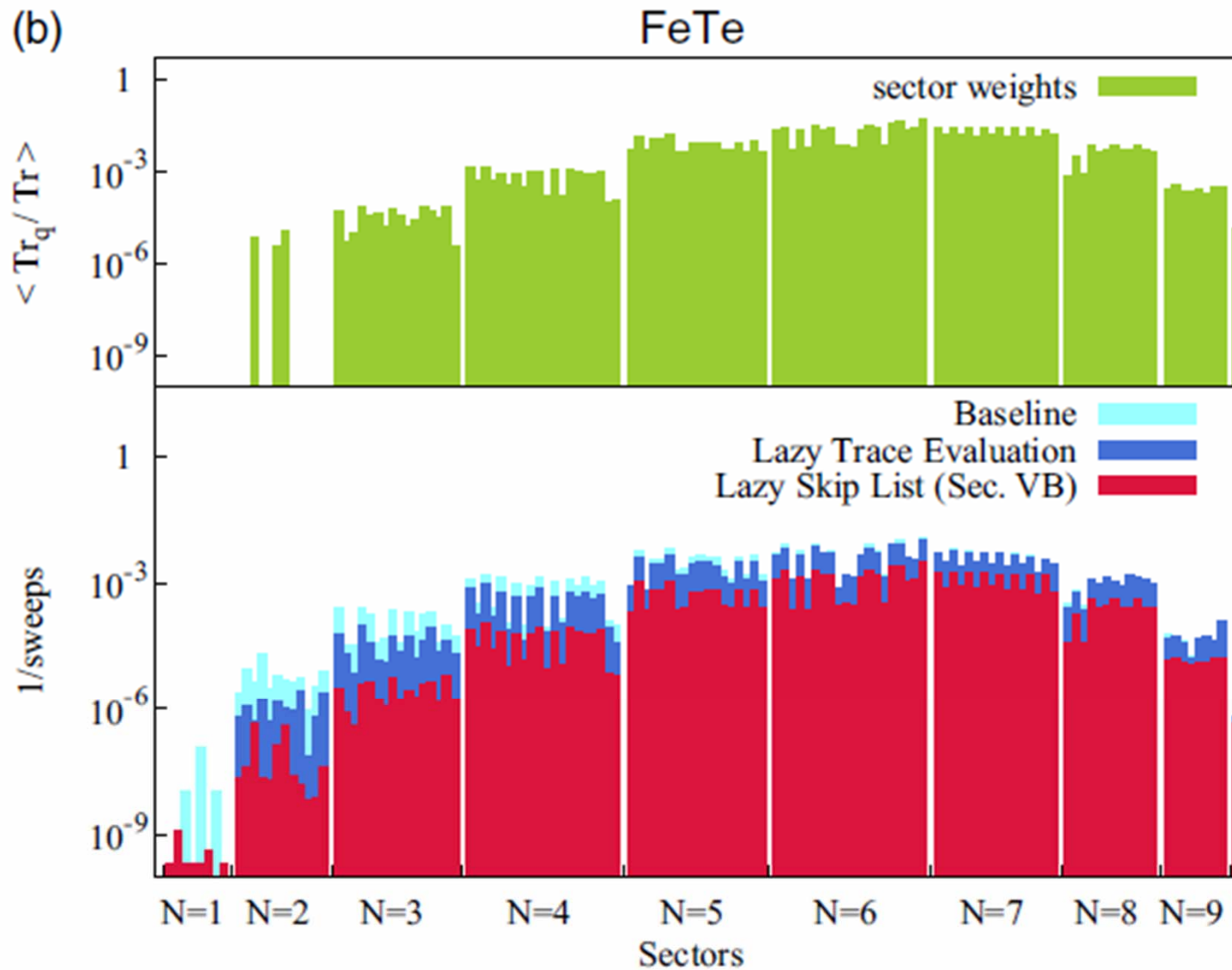
Lazy Skip-List: Speedup (beat Moore)



continued



continued



Collaborators



Giovanni Sordi



David Sénéchal



Alexandre Day



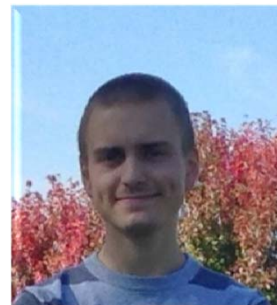
Vincent Bouliane



Patrick Sémon



Kristjan Haule



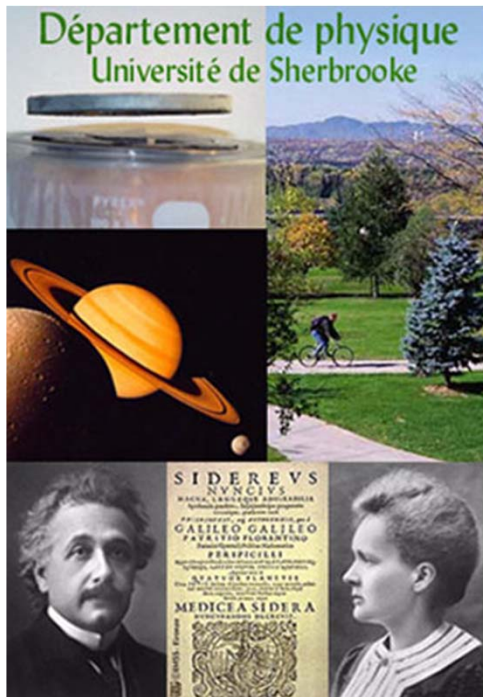
Charles-David Hébert



Chuck-Hou Yee



André-Marie Tremblay



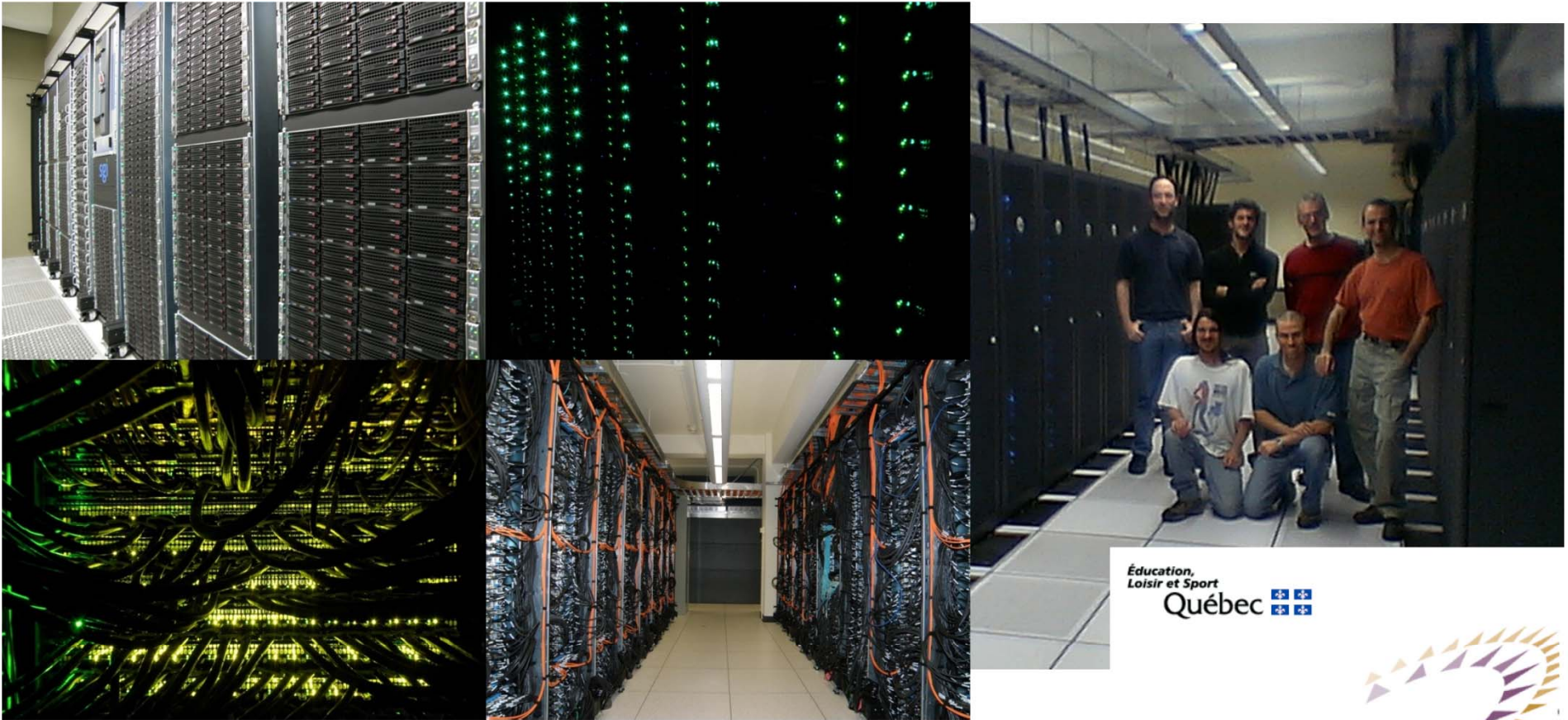
Le regroupement québécois sur les matériaux de pointe



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Mammoth



 **compute + calcul**
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CREATING KNOWLEDGE
DRIVING INNOVATION
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Review: A.-M.S.T. [arXiv: 1310.1481](https://arxiv.org/abs/1310.1481)

Merci

Thank you

A.-M.S. Tremblay

“Strongly correlated superconductivity”

Chapt. 10 : *Emergent Phenomena in Correlated Matter Modeling and Simulation, Vol. 3*, E. Pavarini, E. Koch, and U. Schollwöck (eds.)

Verlag des Forschungszentrum Jülich, 2013