The superfluid density in systems with complex interactions

Presented by

Valéry G. Rousseau

Collaborators:
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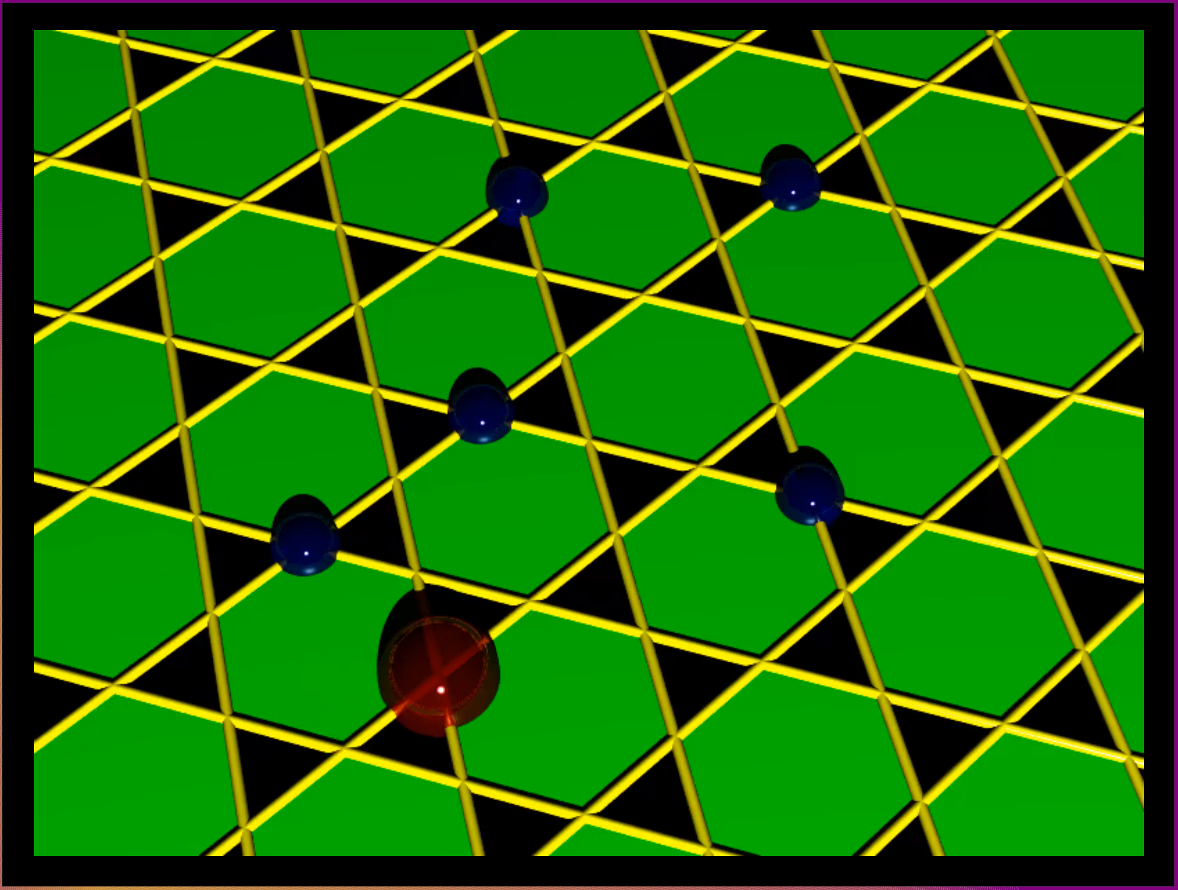
- Mark Jarrell
- Juana Moreno
- George Batrouni
- Richard Scalettar



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Content

- What does "complex interactions" stand for?
- Examples of Hamiltonians with complex interactions recently studied.
- The superfluid density: Avoiding misconceptions.

What does "complex interactions" stand for?

Particles in interaction are often described by Hamiltonians that take the form

$$\hat{\mathcal{H}} = -\sum_{s} t_s \sum_{\langle i,j \rangle} \left(a^{\dagger}_{i,s} a_{j,s} + H.c. \right) + \hat{\mathcal{V}}$$

where *i* and *j* run over first-neighboring sites, *s* runs over all species of particles, and ***V*** depends only on occupation numbers.

In other words, the non-diagonal part of the Hamiltonian is assumed to be the usual kinetic operator.

But...

Many Hamiltonians of interest do not fit with this form!

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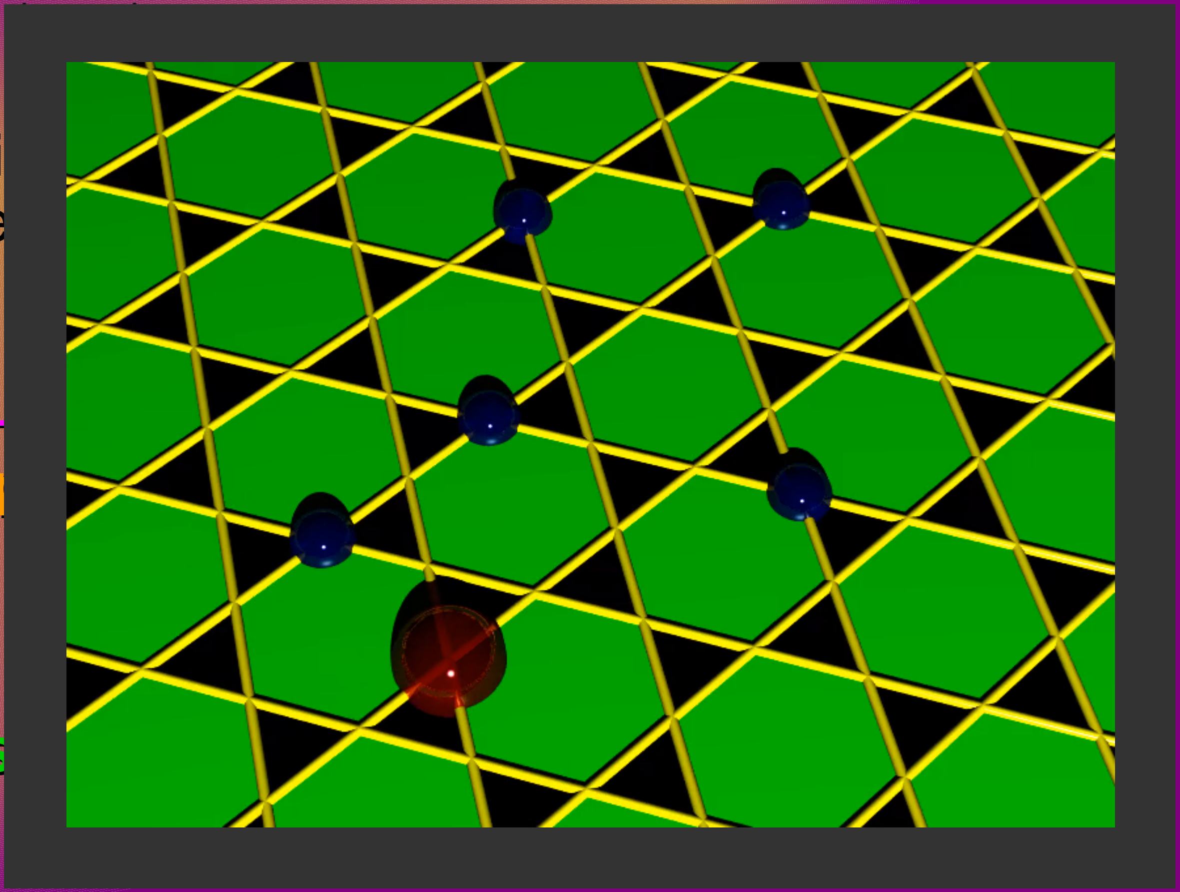

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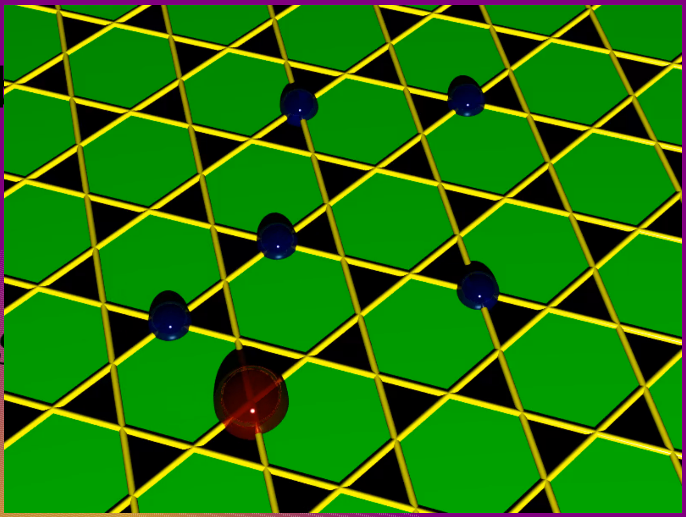
$$\hat{\mathcal{H}} = -\sum_{s} t_s \sum_{\langle i,j\rangle} \left(a^{\dagger}_{i,s} a_{j,s}\right.$$



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Example

If we are interested in the BCS-BEC crossover in a lattice, we may consider the following Hamiltonian:

$$\begin{aligned}\hat{\mathcal{H}} = & -t_a \sum_{\langle i,j\rangle} (a_i^\dagger a_i + H.c.) - t_m \sum_{\langle i,j\rangle} (m_i^\dagger m_i + H.c.) \\ & + U_{aa} \sum_i \hat{n}_i^a (n_i^a - 1) + U_{mm} \sum_i \hat{n}_i^m (n_i^m - 1) + U_{am} \sum_i \hat{n}_i^a n_i^m \\ & + g \sum_i (a_i^\dagger a_i^\dagger m_i + H.c.)\end{aligned}$$

The superfluid density: Avoiding misconceptions

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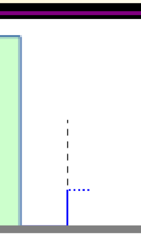
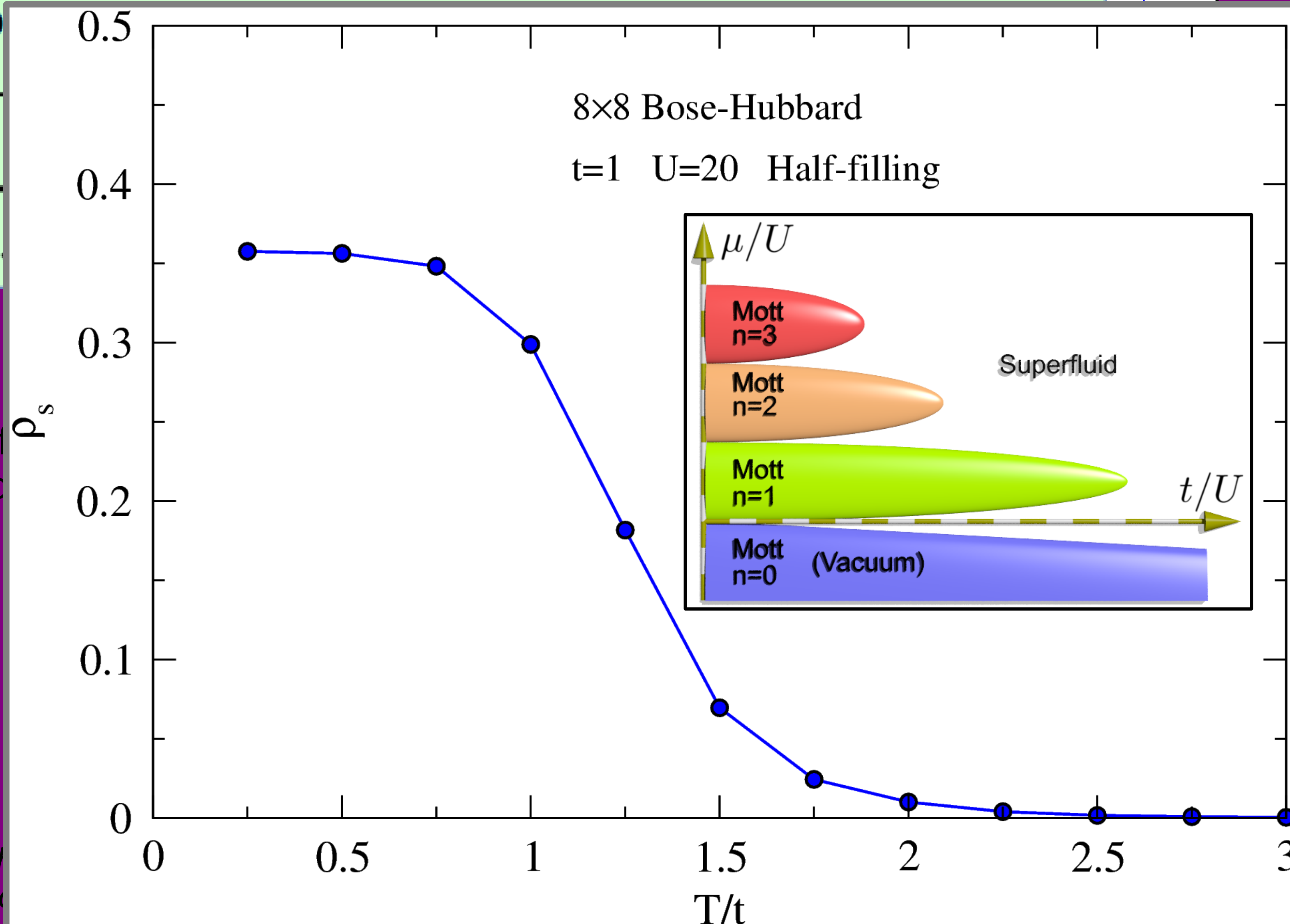


Example

Two-dimensio

$$\hat{\mathcal{H}} = -t \sum_{\langle i,}$$



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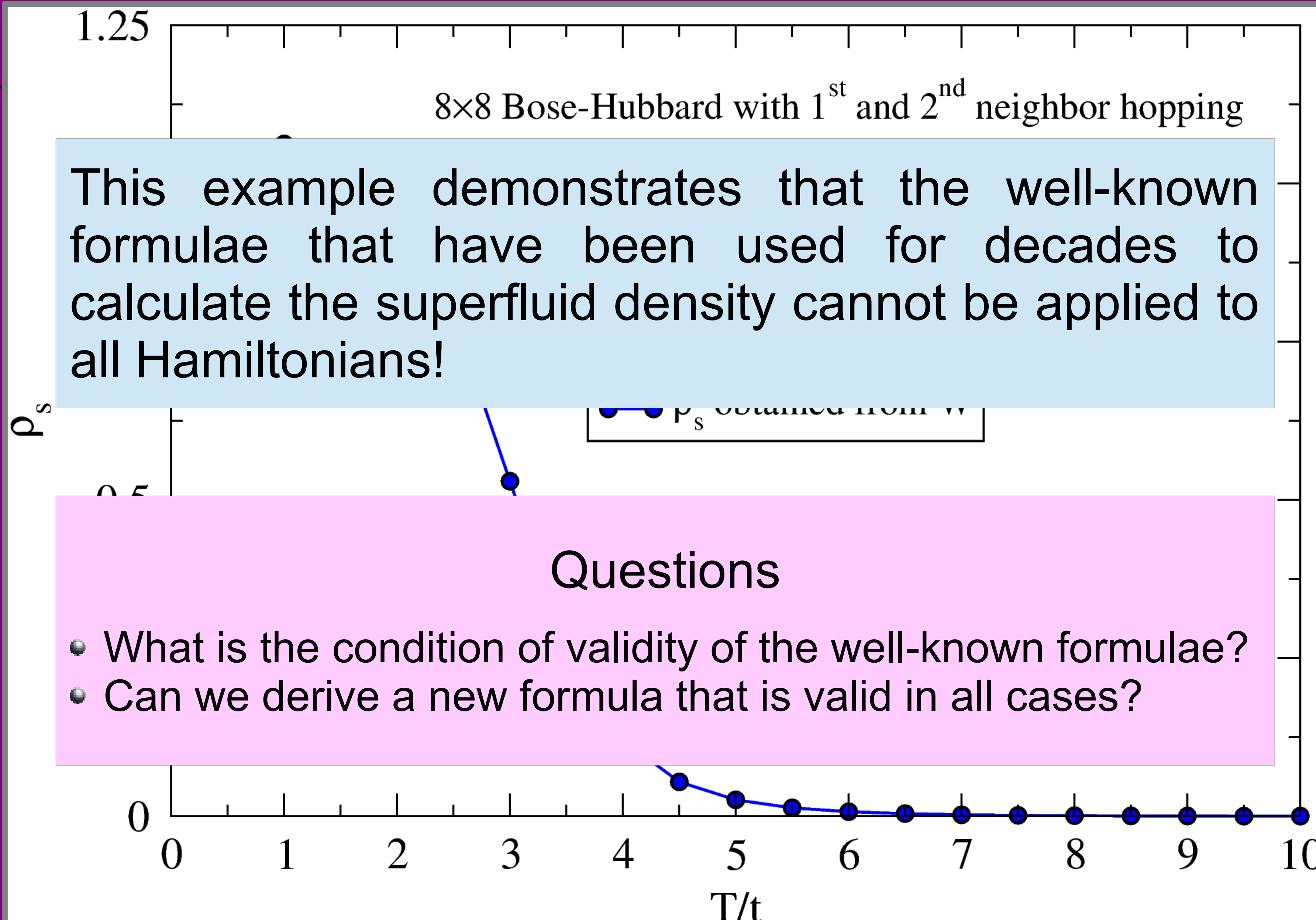
References

[1] M. E. Fis
[2] E. L. Poll

The superfluid density: Avoiding misconceptions

Two-dimensional Bose-Hubbard model with *second-neighbor* hopping at half-filling:

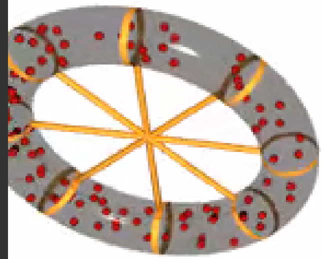


This example demonstrates that the well-known formulae that have been used for decades to calculate the superfluid density cannot be applied to all Hamiltonians!

Questions

- What is the condition of validity of the well-known formulae?
- Can we derive a new formula that is valid in all cases?

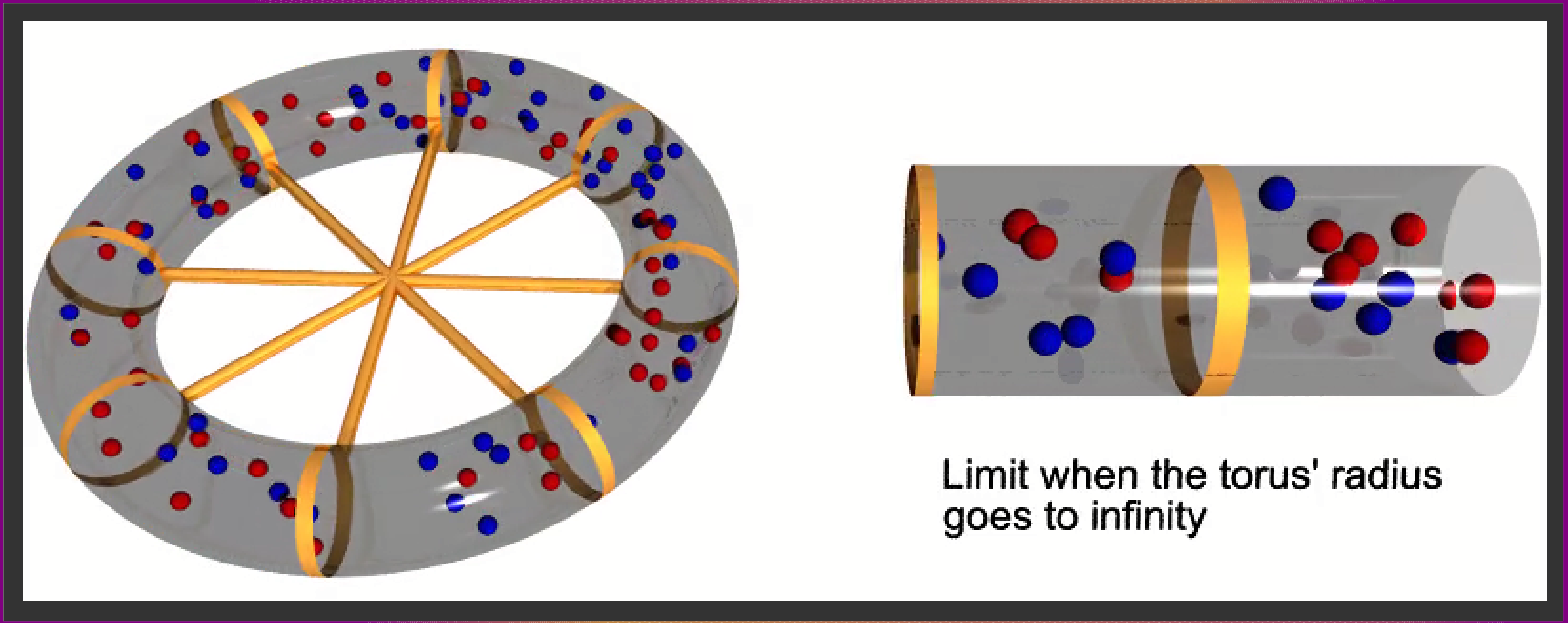
Experimental detection of superfluidity



Thought experiment and correspondence principle

<u>**Thought experiment:**</u> We idealize the real experiment by taking the limit when the torus' radius R goes to infinity. The system then becomes equivalent to a fluid enclosed in a cylinder of length $2\pi R$.

<u>Outcome</u>: A superfluid does not respond to a Galilean boost.

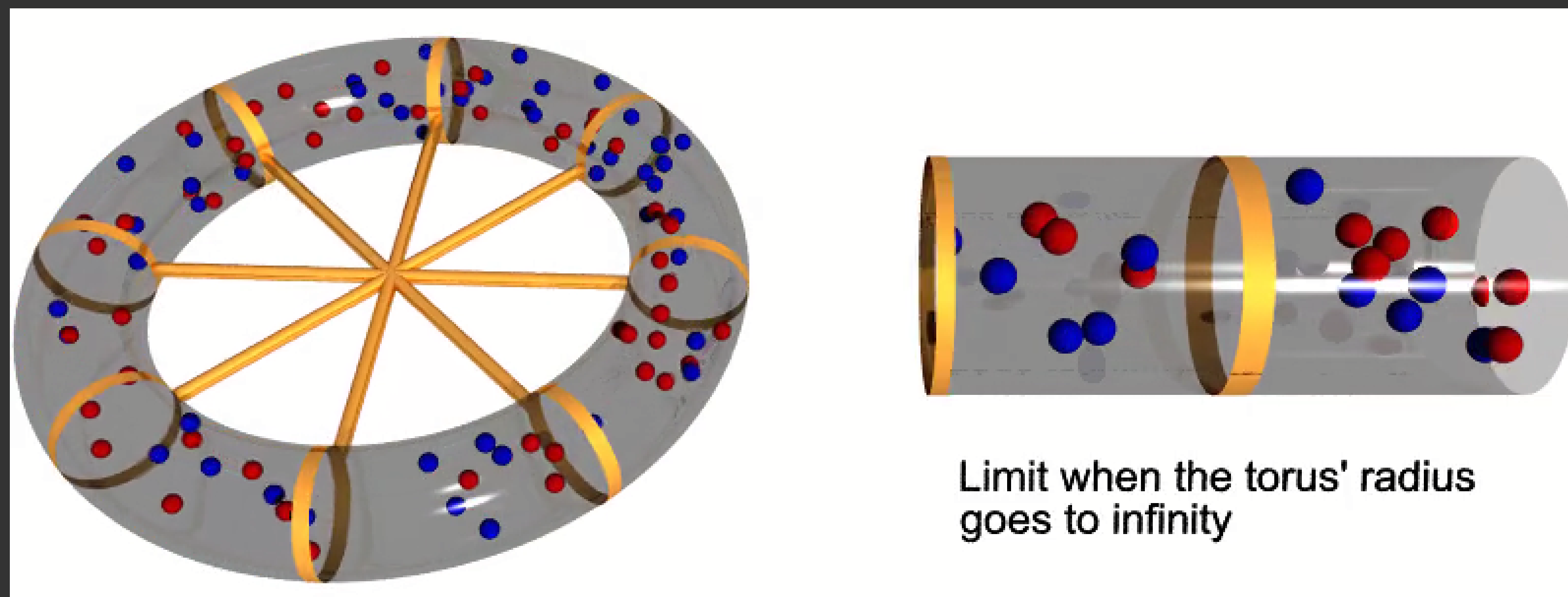


<u>**Correspondence principle:**</u> The behavior of systems described by Quantum Mechanics must reproduce Classical Physics in the macroscopic limit.

Thought experiment and correspondence principle

Galilean boost operator: $\hat{\mathcal{U}} = e^{-i\frac{m}{\hbar}\vec{v}\cdot\vec{\mathcal{R}}}$

$$\hat{\mathcal{U}}^{\dagger}\vec{\mathcal{P}}\hat{\mathcal{U}} = \vec{\mathcal{P}} - m\vec{v}\hat{\mathcal{N}} \qquad \hat{\mathcal{U}}^{\dagger}\vec{\mathcal{R}}\hat{\mathcal{U}} = \vec{\mathcal{R}}$$

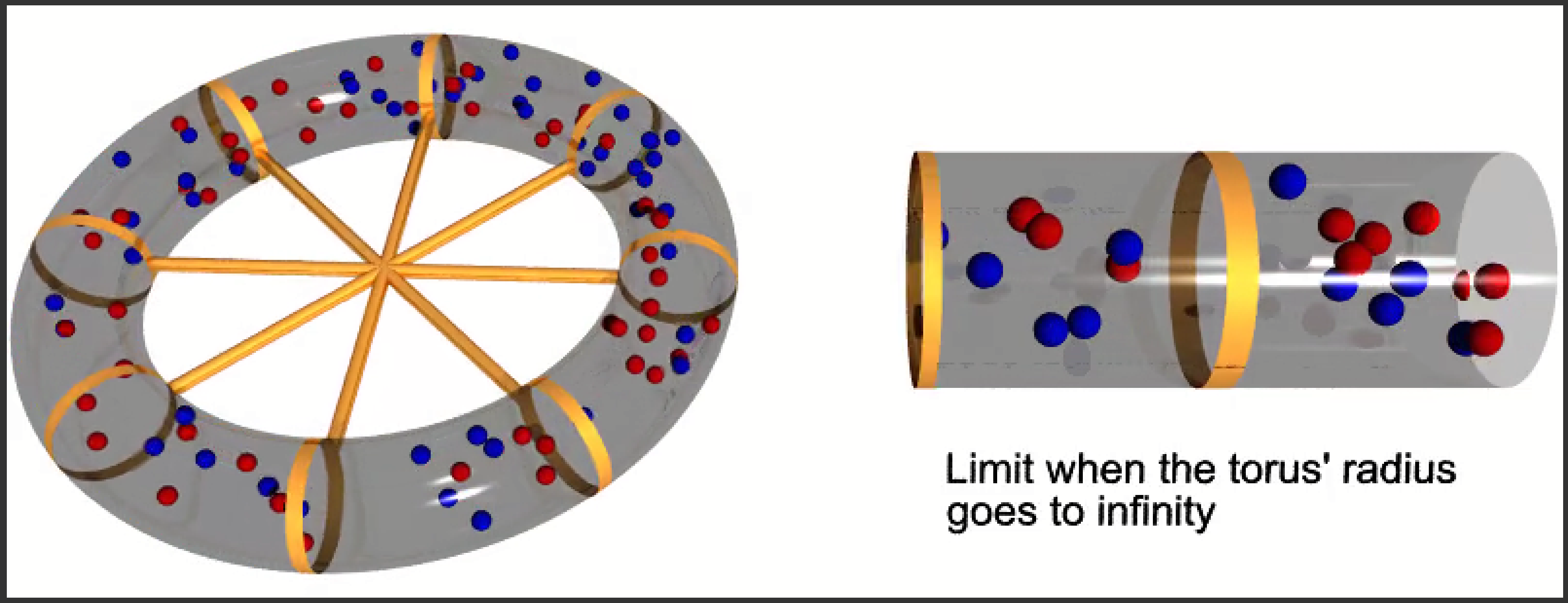


Correspondence principle: The behavior of systems described by Quantum Mechanics must reproduce Classical Physics in the macroscopic limit.

Thought experiment and correspondence principle

Quantum average of the total momentum in the lab frame:

$$\langle \vec{\mathcal{P}} \rangle = \frac{1}{\mathcal{Z}_o} \mathrm{Tr}\ \hat{\mathcal{U}} \hat{\vec{\mathcal{P}}} \hat{\mathcal{U}}^\dagger e^{-\beta \hat{\mathcal{H}}_o}$$



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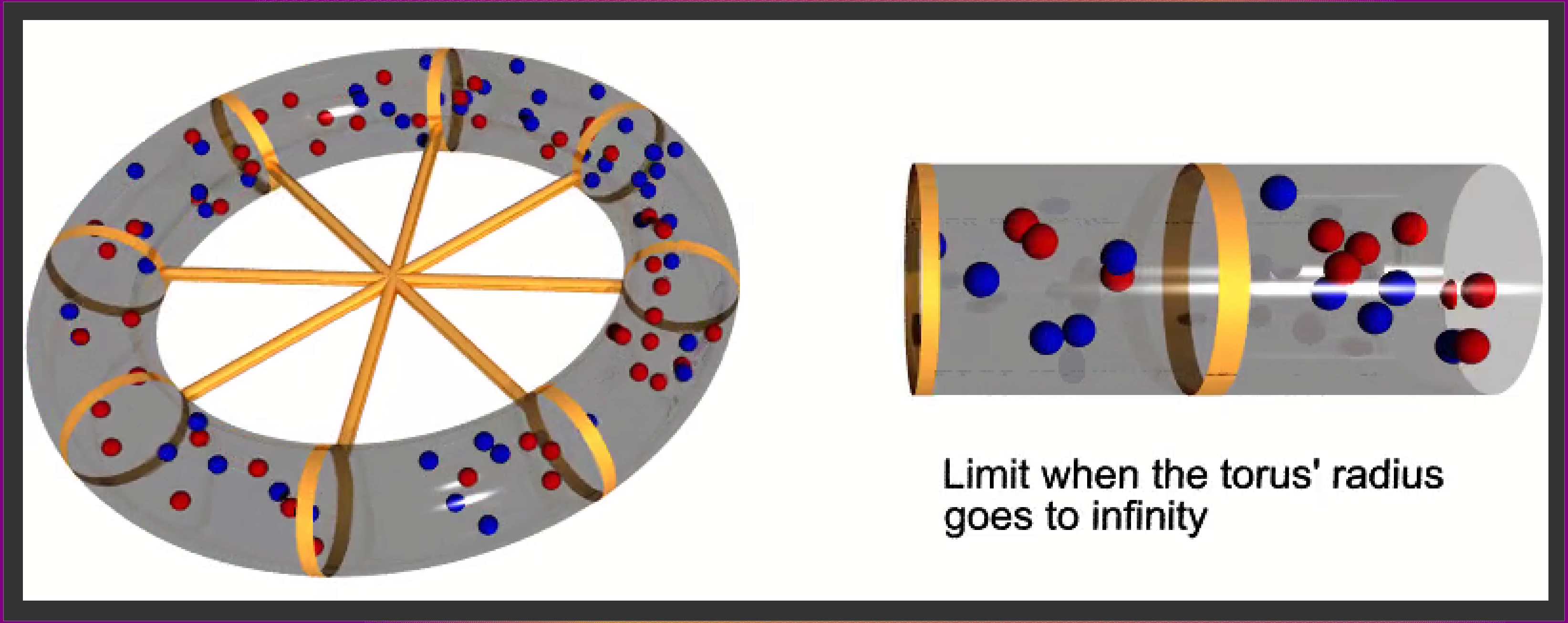

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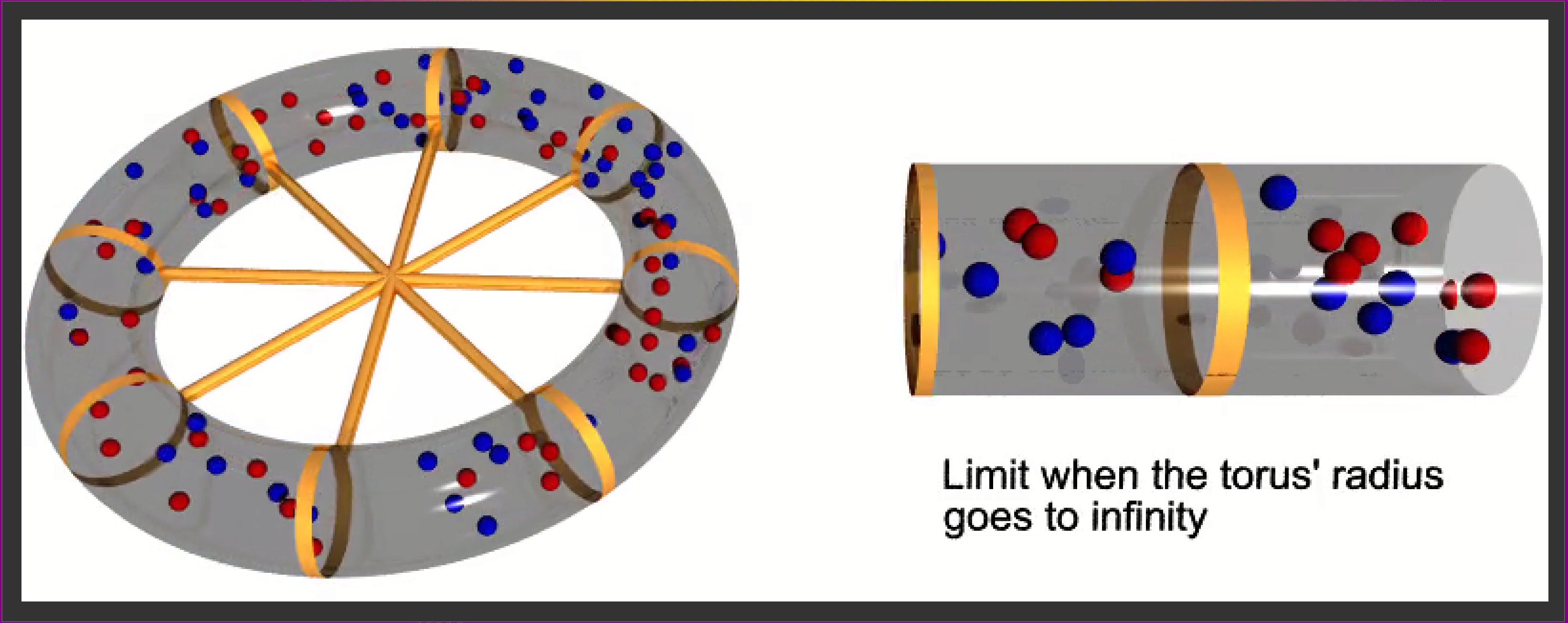
Thought experiment and correspondence principle

$$\frac{1}{\mathcal{Z}_o}\mathrm{Tr}\ \vec{\mathcal{P}}e^{-\beta\hat{\mathcal{U}}^\dagger\hat{\mathcal{H}}_o\hat{\mathcal{U}}} \underbrace{=}_{\text{Correspondence principle}} \text{Classical momentum} \underbrace{=}_{\text{Thought experiment}} \rho_n\Omega\vec{v}$$

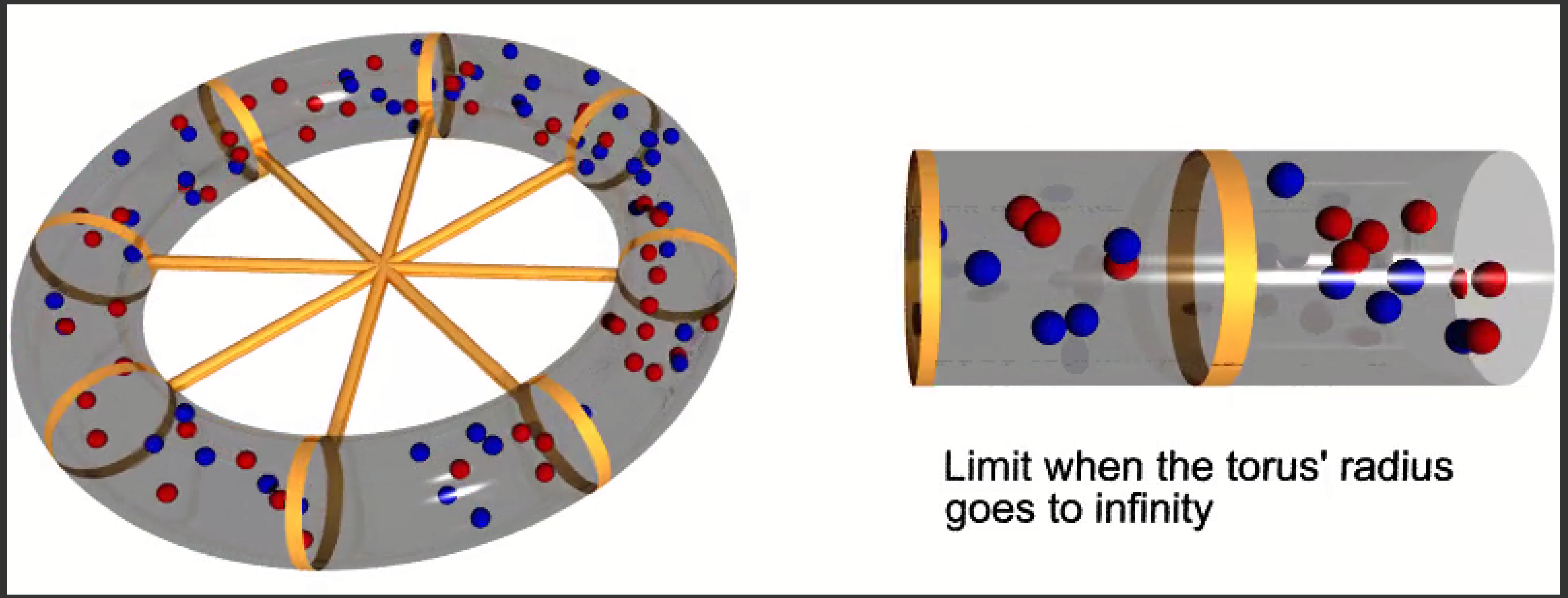


Thought experiment and correspondence principle

Normal density: $$\rho_n = -i\frac{m}{\hbar\Omega d}\left\langle \vec{\mathcal{P}}\cdot\int_0^\beta e^{\tau\hat{\mathcal{H}}_o}\big[\vec{\mathcal{R}},\hat{\mathcal{H}}_o\big]e^{-\tau\hat{\mathcal{H}}_o}d\tau\right\rangle$$

Superfluid density: $$\rho_s = \rho + i\frac{m}{\hbar\Omega d}\left\langle \vec{\mathcal{P}}\cdot\int_0^\beta e^{\tau\hat{\mathcal{H}}_o}\big[\vec{\mathcal{R}},\hat{\mathcal{H}}_o\big]e^{-\tau\hat{\mathcal{H}}_o}d\tau\right\rangle$$

V.G. Rousseau, Phys. Rev. B 90, 134503 (2014).
“Superfluid density in continuous and discrete spaces: Avoiding misconceptions”



Test of the new formula

It can be shown that Fisher *et al.* and Pollock & Ceperley's formulae can be written in terms of momentum correlations:

$$\rho_s = \rho - \frac{1}{\Omega d}\left\langle \vec{\mathcal{P}} \cdot \int_0^\beta e^{\tau\hat{\mathcal{H}}_o}\vec{\mathcal{P}}e^{-\tau\hat{\mathcal{H}}_o}d\tau \right\rangle$$

The new formula,

$$\rho_s = \rho + i\frac{m}{\cdots}\left\langle \vec{\mathcal{P}} \cdot \int_0^\beta e^{\tau\hat{\mathcal{H}}_o}[\vec{\mathcal{R}},\hat{\mathcal{H}}_o]e^{-\tau\hat{\mathcal{H}}_o}d\tau \right\rangle$$

This condition is systematically satisfied for Hamiltonians with no complex interactions

reproduce

$$\left[\vec{\mathcal{R}},\hat{\mathcal{H}}_o\right] = i\frac{\hbar}{m}\vec{\mathcal{P}}$$

This is the condition that the Hamiltonian must satisfy for the old formulae to be applicable.

V.G. Rousseau, Phys. Rev. B 90, 134503 (2014).
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Test of the new formula

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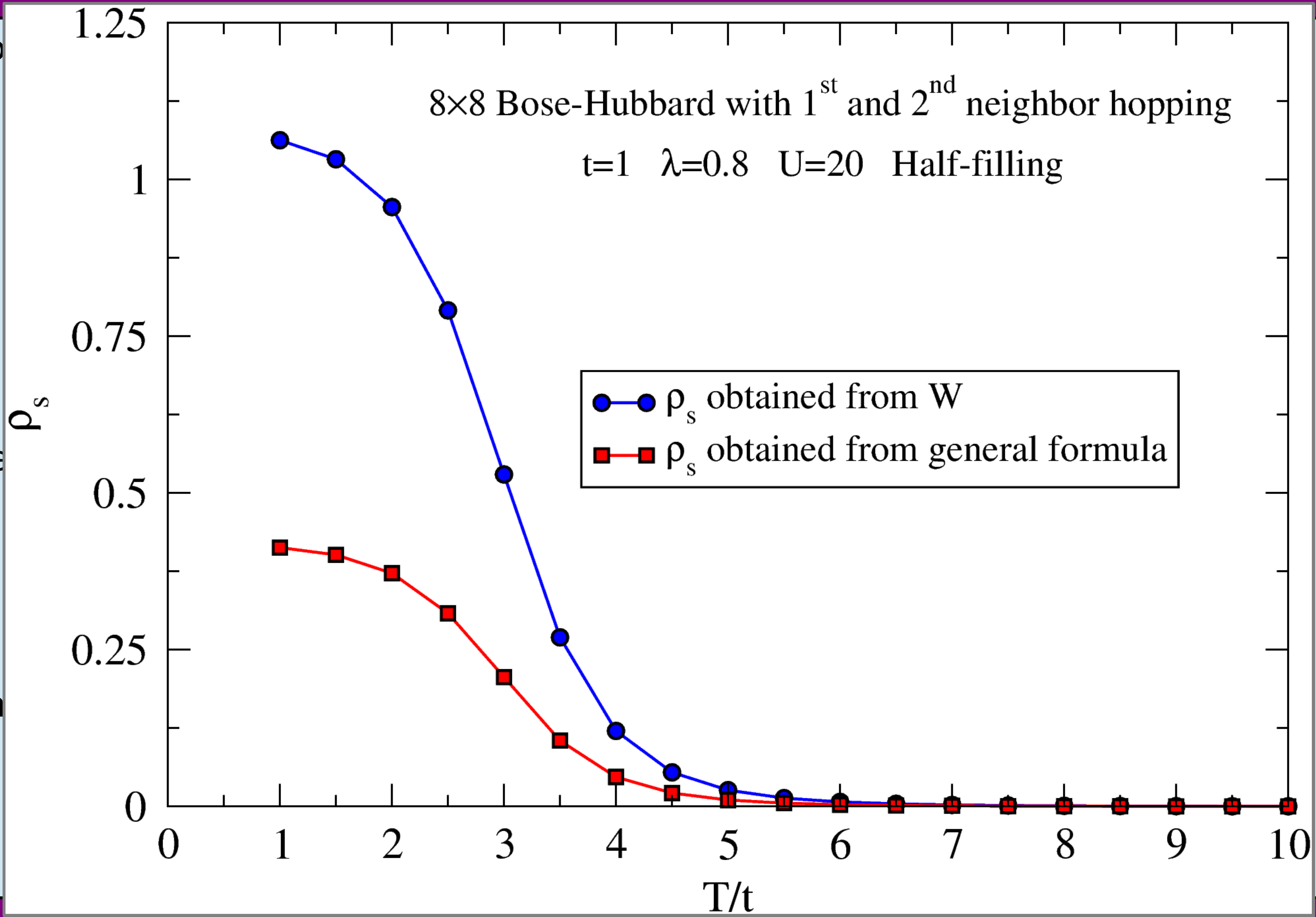

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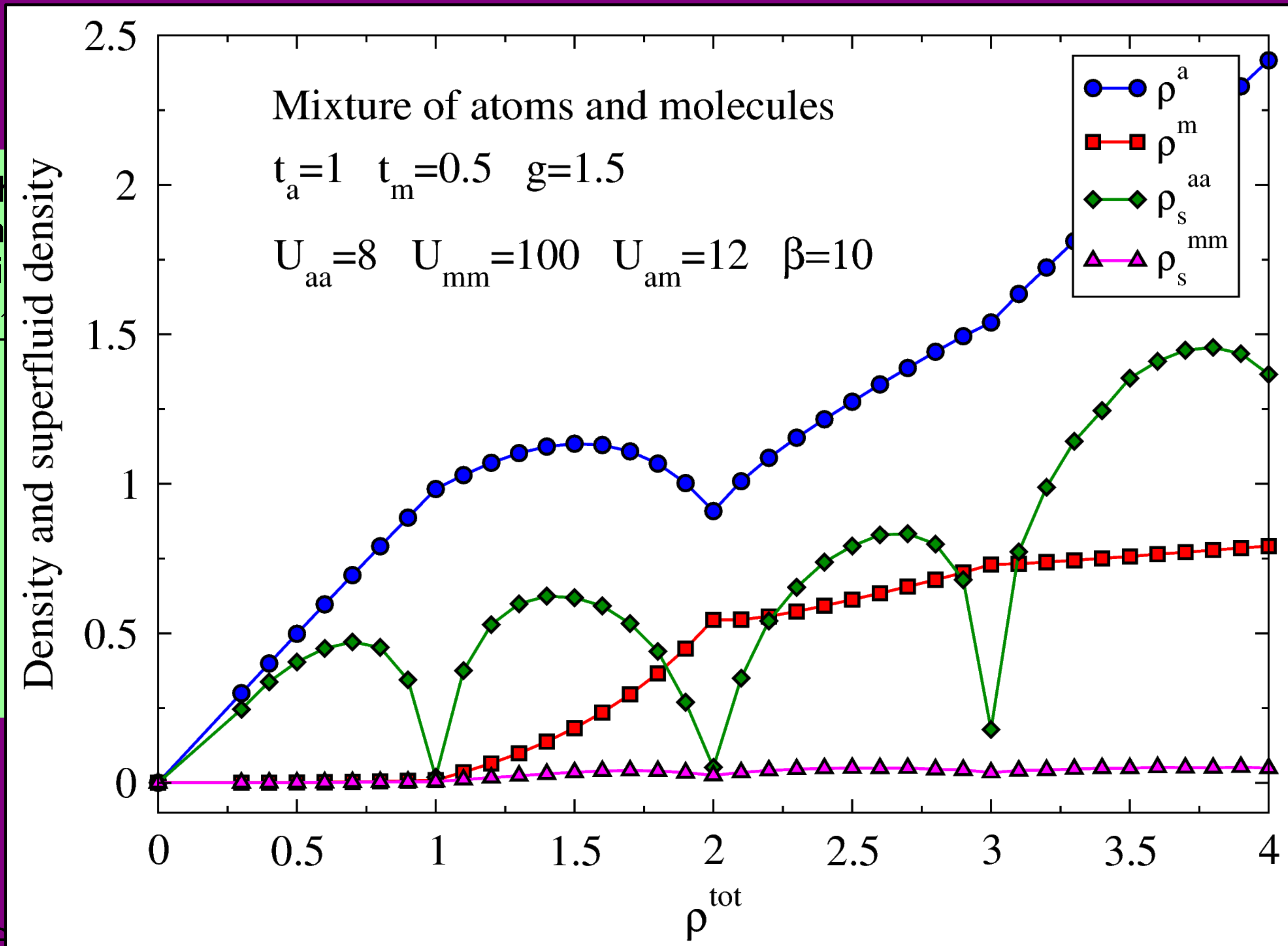

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"Superfluid density in continuous and discrete spaces: Avoiding misconceptions"

Conclusion

The ***superfluid density*** is an important quantity that allows us to characterize various phases encountered in Condensed Matter physics and in quantum gases. However, the well-known formulae used for the calculation of the superfluid density are not applicable in all cases.

Based on a ***thought experiment*** and the ***Correspondence principle***, I derived a general formula for the superfluid density that does not involve any assumption on the form of the Hamiltonian:

$$\rho_s = \rho + i\frac{m}{\hbar\Omega d}\Big\langle \vec{\mathcal{P}}\cdot\int_0^\beta e^{\tau\hat{\mathcal{H}}_o}\big[\vec{\mathcal{R}},\hat{\mathcal{H}}_o\big]e^{-\tau\hat{\mathcal{H}}_o}d\tau\Big\rangle$$

I showed that this new formula gives consistent results for a model with second-nearest neighbor hopping, where the old formulae failed. The impact of this new formula on the ongoing research should be important, given the fact that Hamiltonians with complex interactions are currently under intensive investigations.

"Superfluid density in continuous and discrete spaces: Avoiding misconceptions"
V.G. Rousseau, Phys. Rev. B 90, 134503 (2014).

All QMC results presented in this talk were obtained with the ***SGF*** algorithm.
V.G. Rousseau, Phys. Rev. E 78, 056707 (2008).