

Connecting real materials to low-energy effective
Hamiltonian: applications of symmetry-
respecting Wannier functions

Wei Ku (顧 威)

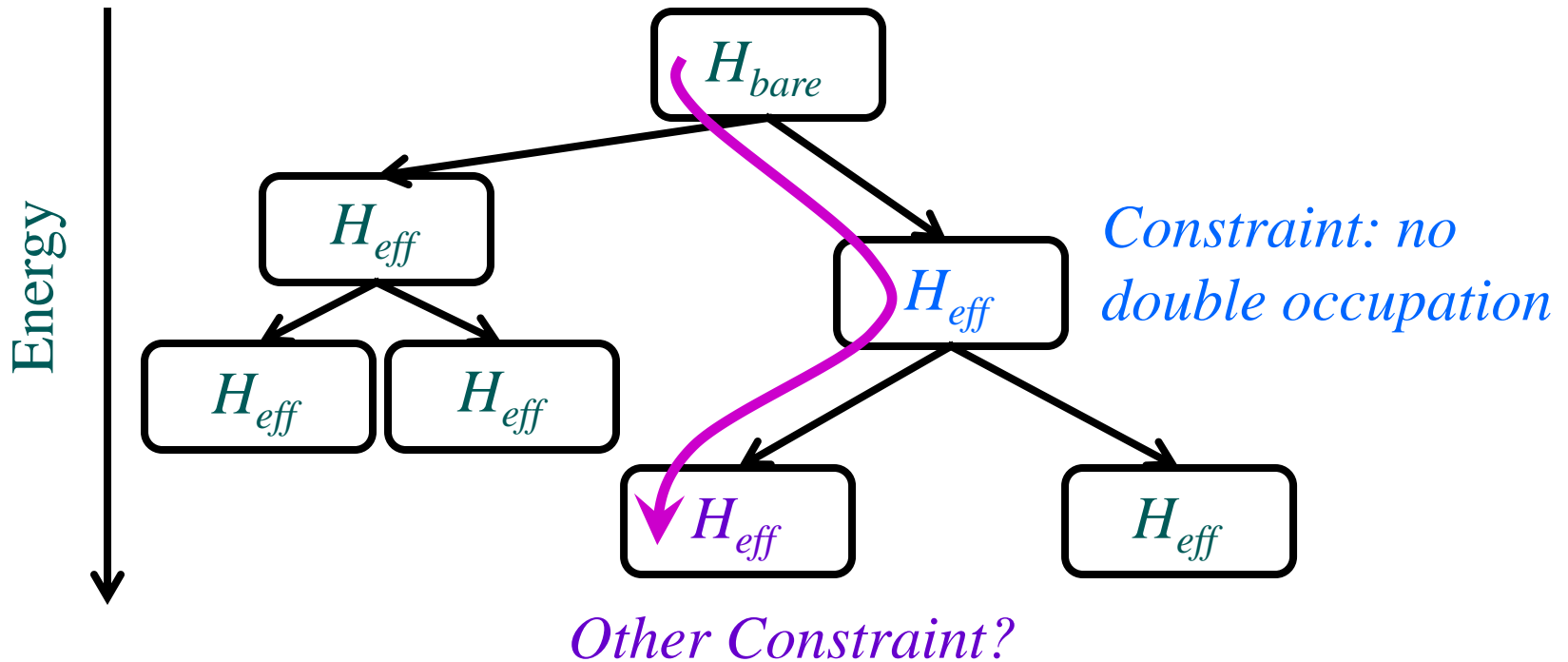
Brookhaven National Lab & SUNY Stony Brook



Outlines

- ➔ ● Simple illustrations:
 - Orbital order in LaMnO_3
 - Gapless CDW in TaSe_2
 - Ferro-orbital order in Fe-SCs
- Interacting problems:
 - Local excitations
 - Propagation and decay of local excitations
- Effects of disordered impurities in Fe-SCs:
 - Substitution of Fe: doping or not?
 - Fe vacancy: “violation” of Luttinger theorem
 - Ru substitution: realization of superdiffusion

RG perspective: bifurcation of many-body systems

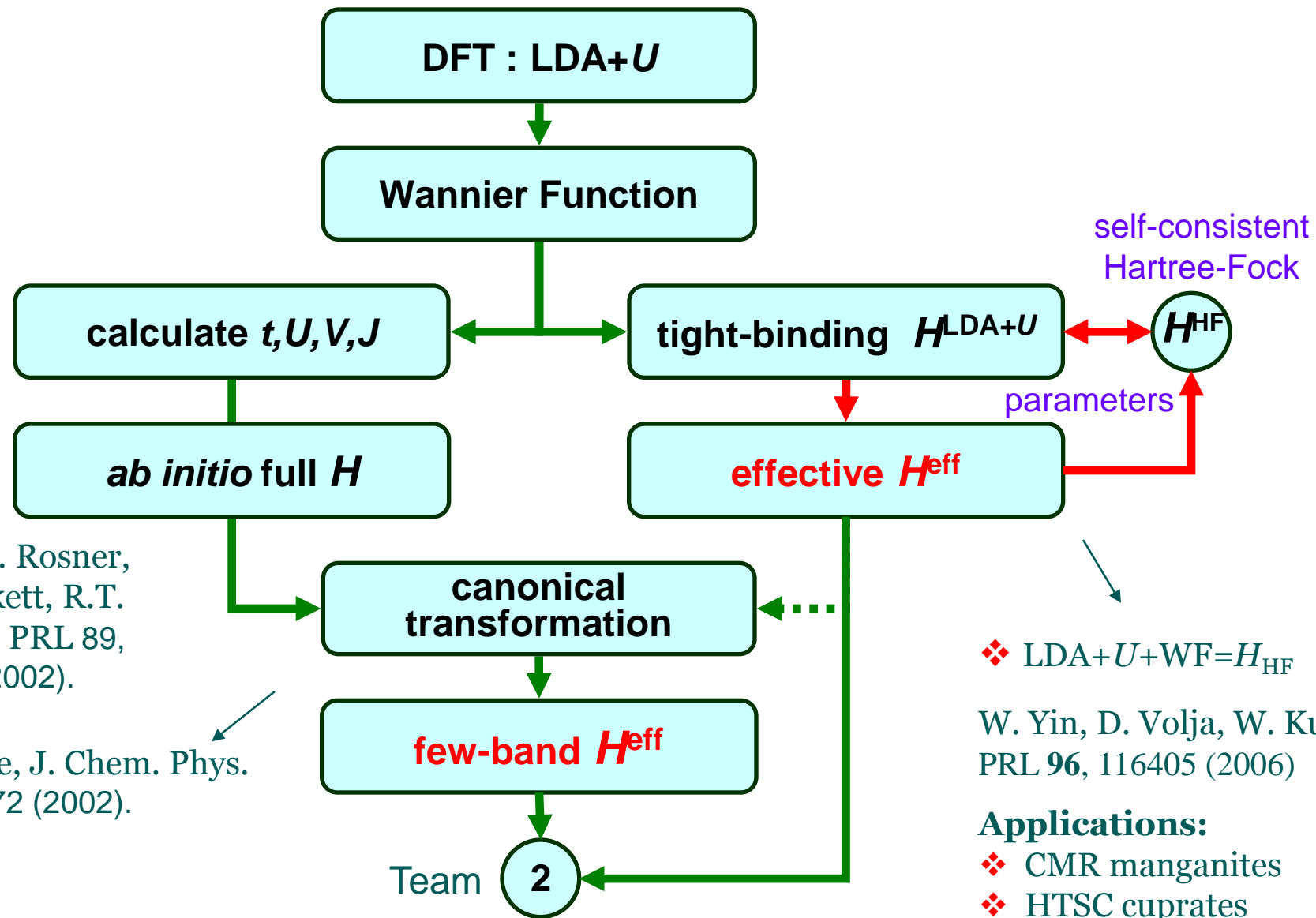


- Lagrangian: integrate out high-energy sectors

vs.

- Hamiltonian: block diagonalize low-energy sectors

Direct mapping to effective Hamiltonian



self-consistent
Hartree-Fock

parameters

❖ $\text{LDA}+U+\text{WF}=H_{\text{HF}}$

W. Yin, D. Volja, W. Ku,
PRL **96**, 116405 (2006)

Applications:

- ❖ CMR manganites
- ❖ HTSC cuprates

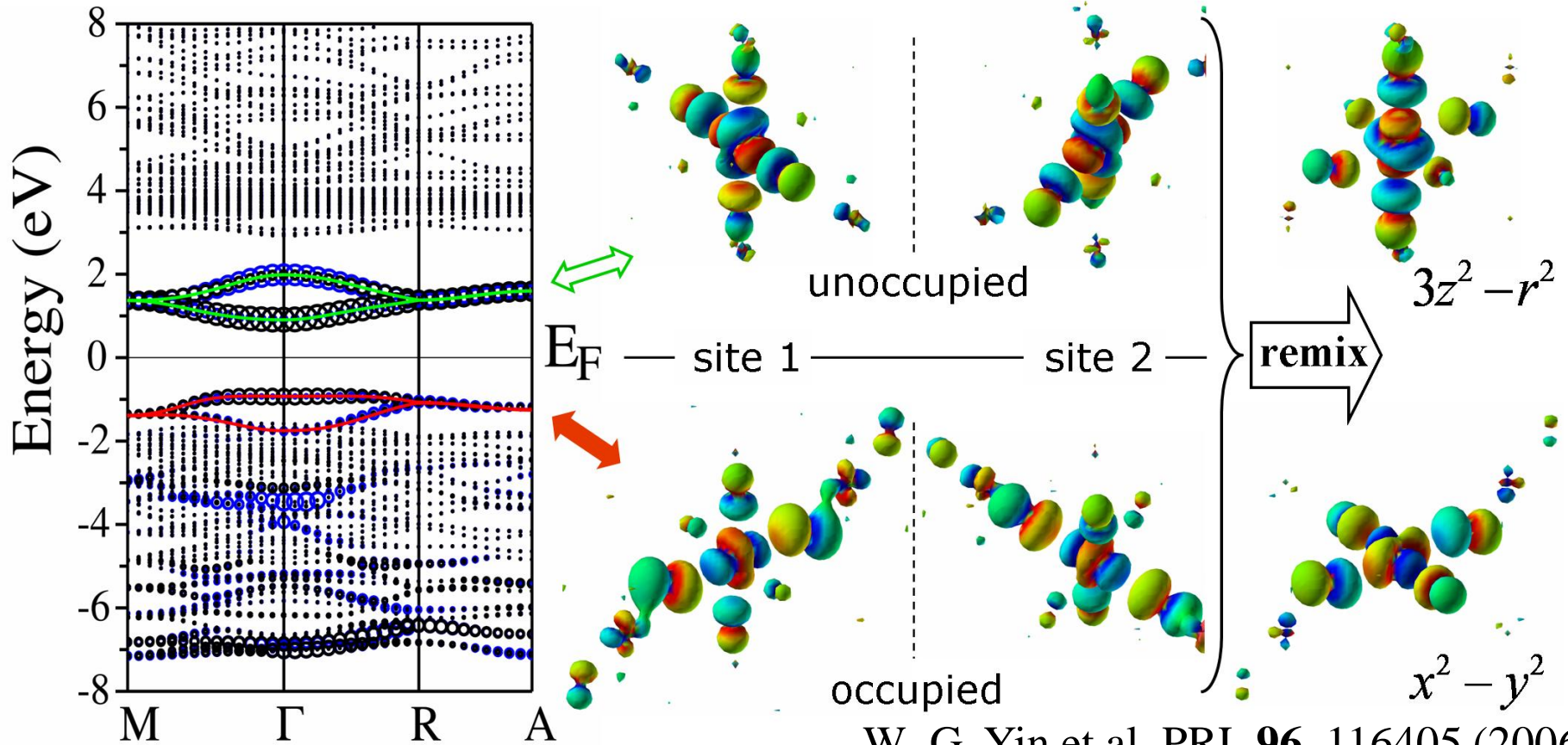
W. Ku, H. Rosner,
W.E. Pickett, R.T.
Scalettar, PRL **89**,
167204 (2002).

S. White, J. Chem. Phys.
117, 7472 (2002).

Team **2**

Orbital Ordering in LaMnO_3

e-e vs. e-l interactions & effective many-body H^{eff}



- Large O component \rightarrow nature of charge transfer gap
- Systematic study of e-e vs. e-l interactions in ordering orbitals

$$H = \sum_{\langle ij \rangle \gamma \gamma'} t_{ij}^{\gamma \gamma'} d_{j\gamma}^\dagger d_{i\gamma} - g \sum_i \vec{T}_i \cdot \vec{Q}_i + \frac{K}{2} \sum_i \vec{Q}_i \cdot \vec{Q}_i + U_{\text{eff}} \sum_i n_{i\uparrow} n_{i\downarrow} - E_z \sum_i T_i^z.$$

pseudo-spin: $T_i^z = \frac{1}{2}(d_{i\uparrow}^\dagger d_{i\uparrow} - d_{i\downarrow}^\dagger d_{i\downarrow}), T_i^x = \frac{1}{2}(d_{i\uparrow}^\dagger d_{i\downarrow} + d_{i\downarrow}^\dagger d_{i\uparrow}).$

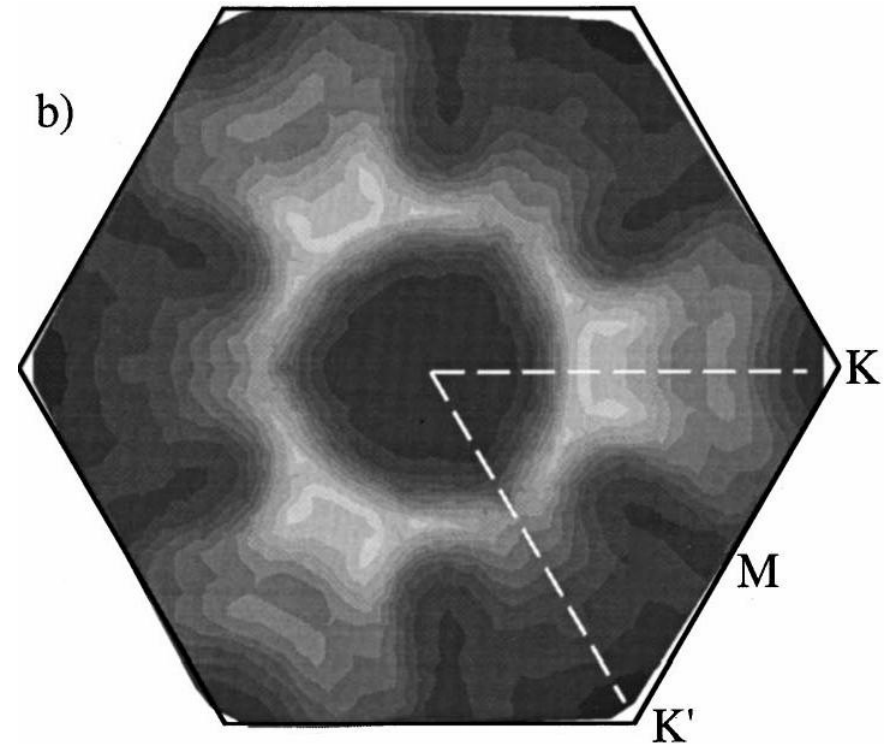
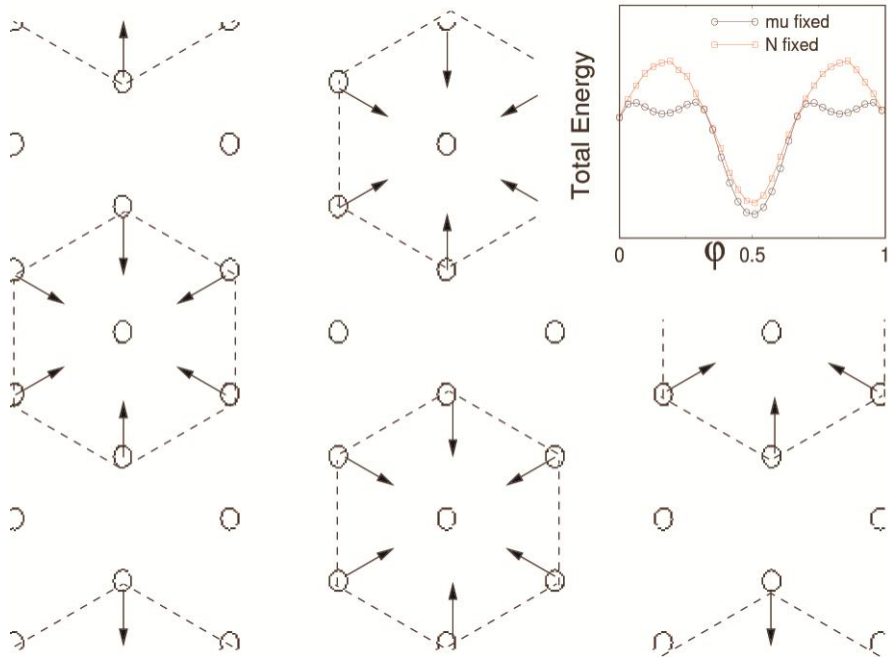


Gapless charge density wave in TaSe₂

R. L. Barnett, A. P., E. Demler, W.-G. Yin, and Wei Ku

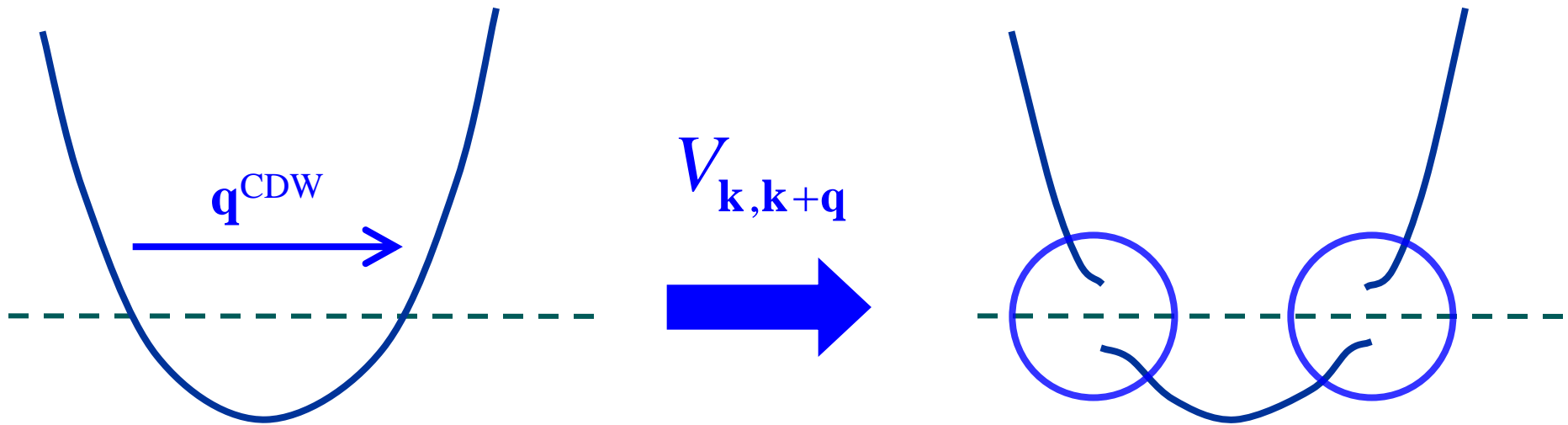
Phys. Rev. Lett. **96**, 026406 (2006)

Gapless Charge-Density Wave in TaSe₂



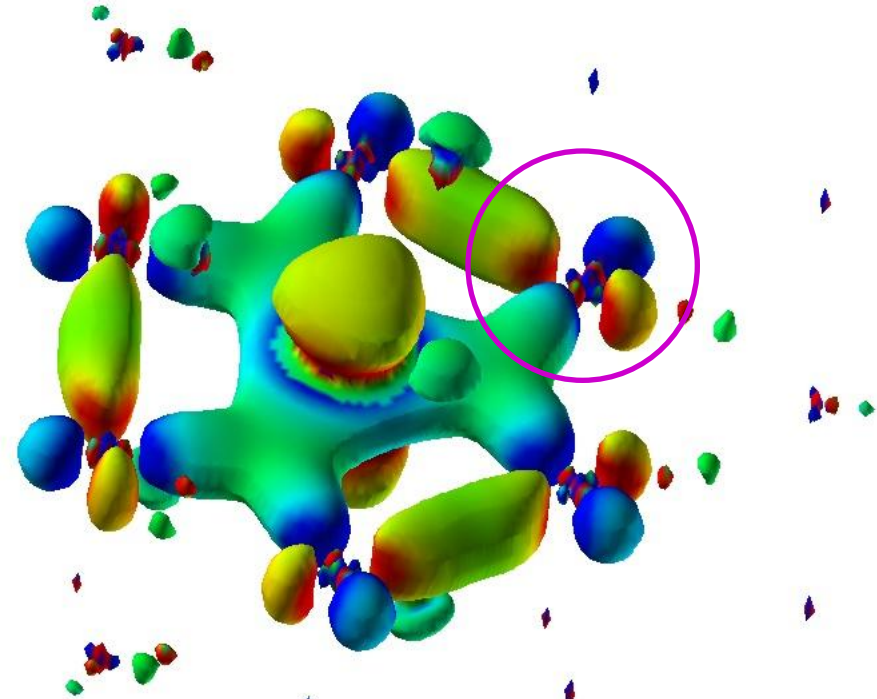
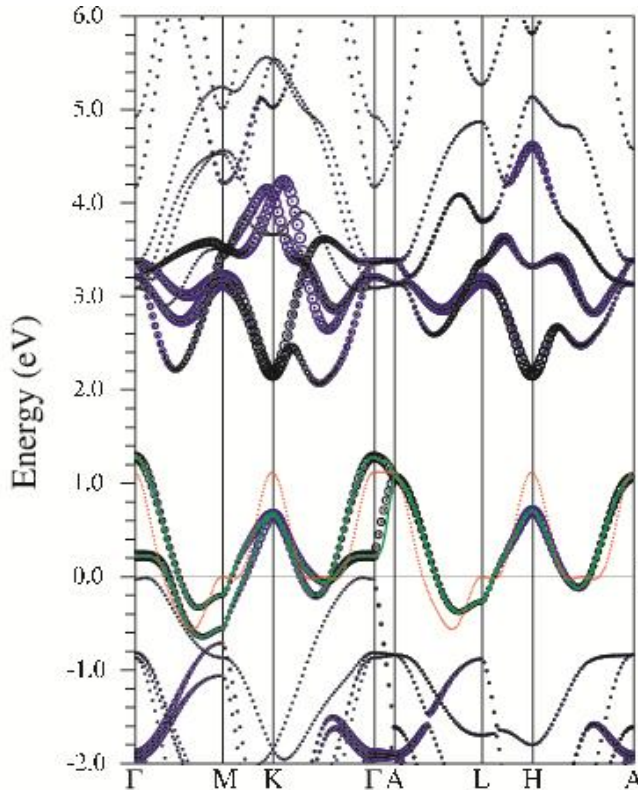
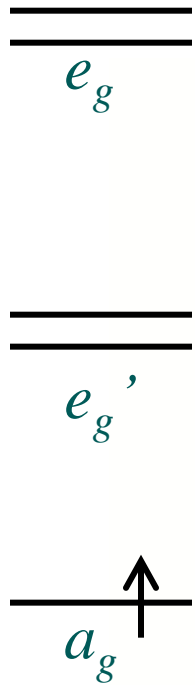
- commensurate CDW
- gapless excitations throughout the whole Fermi surface in CDW phase ?
- Fermi surface Nesting vector too large for q^{CDW} ?

Conventional gapped CDW picture



- Fermi surface instability
 - divergent $\chi(\mathbf{q}^{\text{CDW}}, \omega \rightarrow 0)$
 - nesting preferred
- gap \rightarrow energy gain

Local Picture: Low-Energy Wannier Function



- $3z^2 - r^2$ (a_g) symmetry near E_F , noticeable hybridization with e_g'
- WS in one site contains complete information of the full k-space
- center: a_g symmetry; tail: e_g' symmetry

Surprising Hopping Path

$$t_{Rn,R'n'}^{DFT} = \langle Rn | h^{DFT} | R'n' \rangle$$

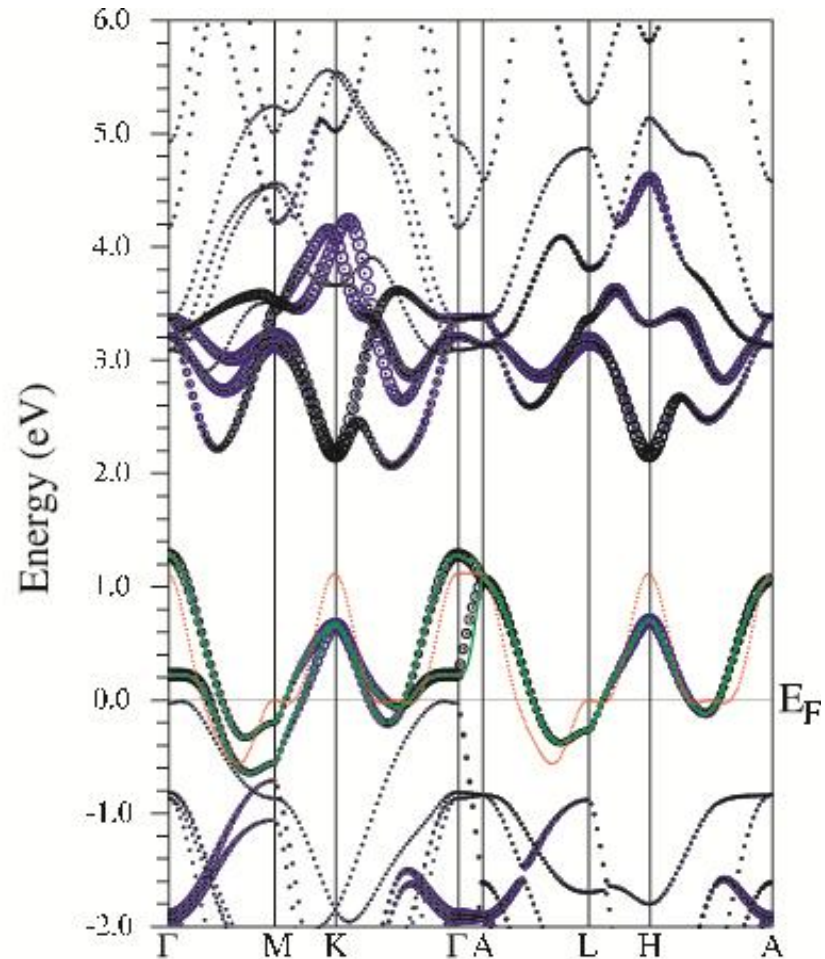
$$t_1 = 38 \text{ (meV)}$$

$$t_2 = 115 \text{ (meV)}$$

$$t_{\perp,1} = 29 \text{ (meV)}$$

$$t_{\perp,2} = 23 \text{ (meV)}$$

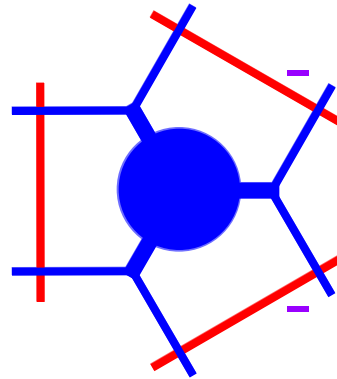
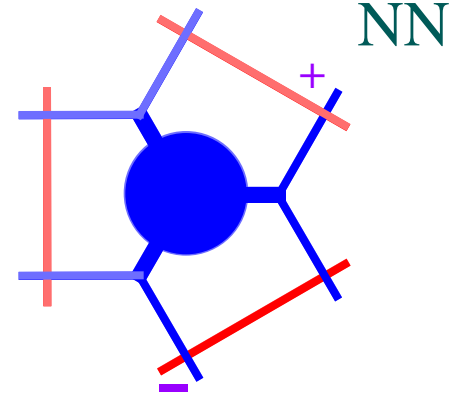
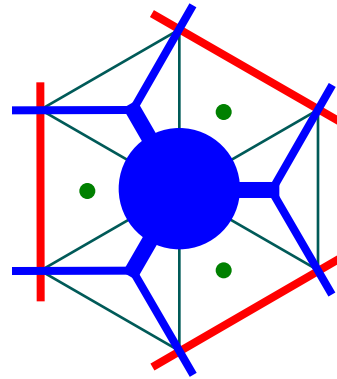
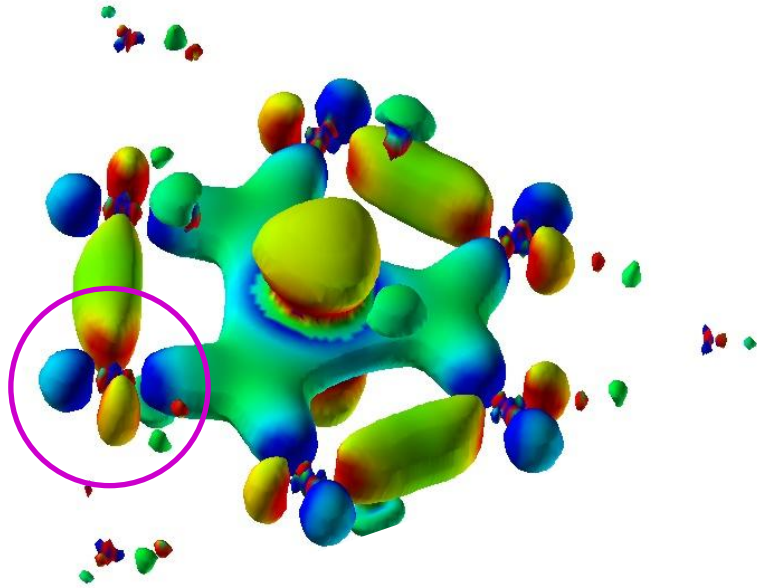
$$t_2 \gg t_1!$$



- perfect tight-binding “fit” to the band structure
- surprising hopping strength to 2nd nearest neighbors

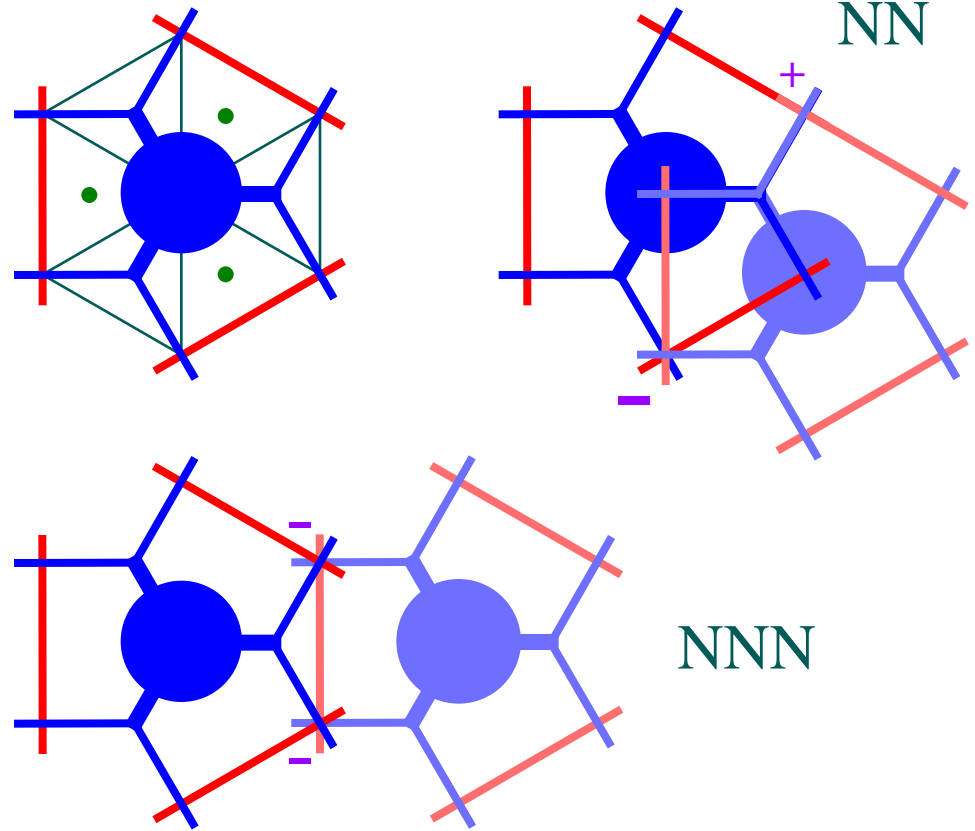
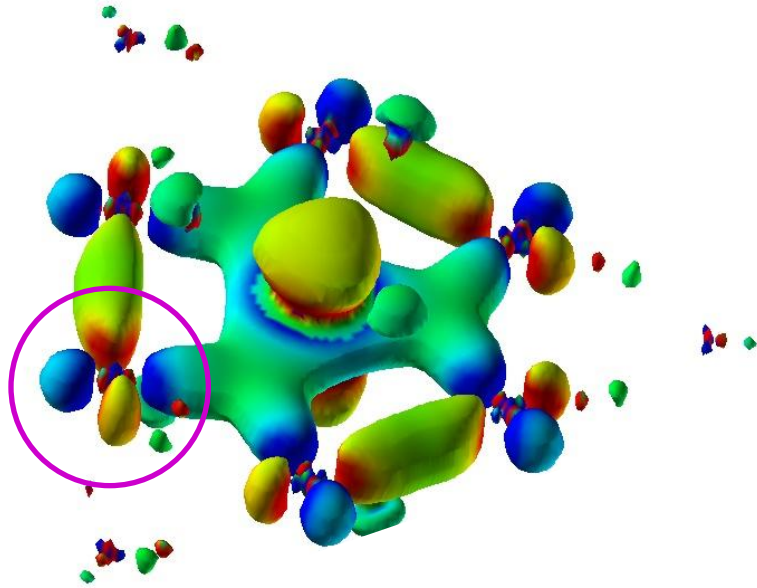


Why $t_2 \gg t_1$?



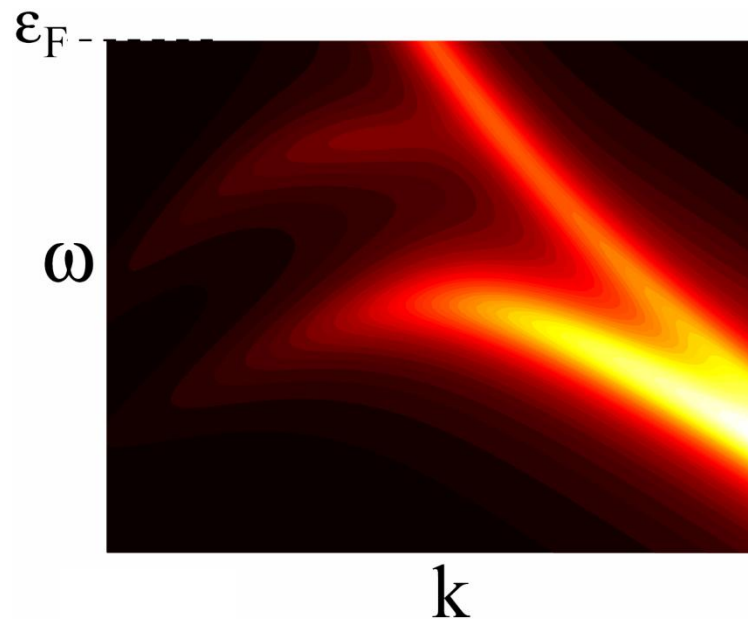
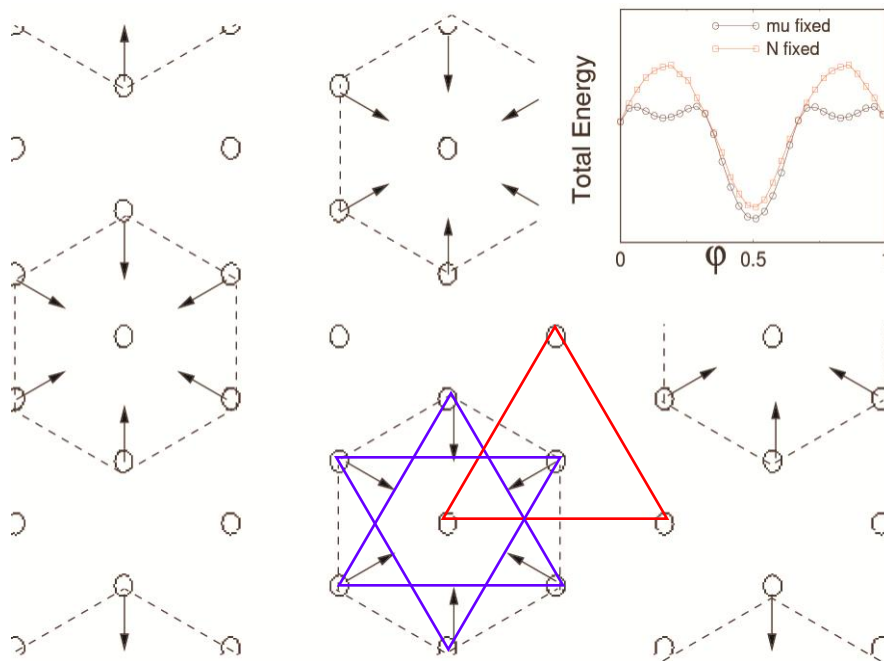
NNN

Why $t_2 \gg t_1$?



- phase interference from e_g ' hybridization tail $\rightarrow t_2 \gg t_1$

Gapless Charge-Density Wave in TaSe₂



- x^2-y^2 hybridization $\rightarrow t_2 \gg t_1 \rightarrow$ decoupling of 3 sublattices
- minimization of tight-binding H against distortion
 - \rightarrow one sublattice undistorted
 - \rightarrow gapless band structure

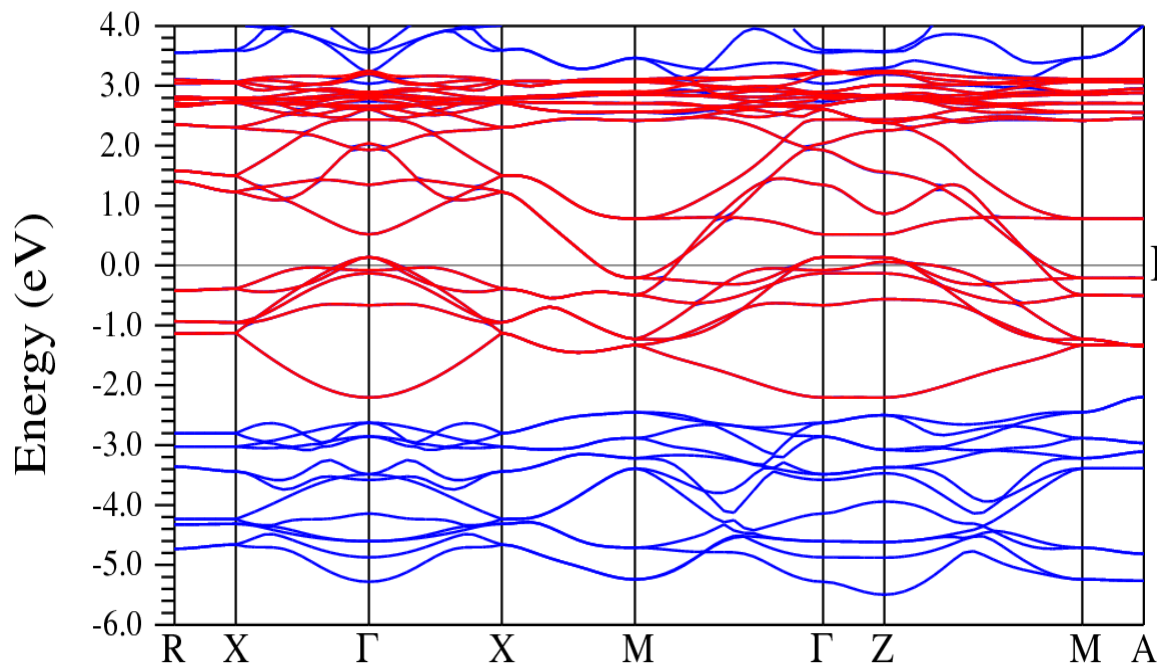


Ferro-orbital order & anisotropic magnetic structure in 1111 (&122)

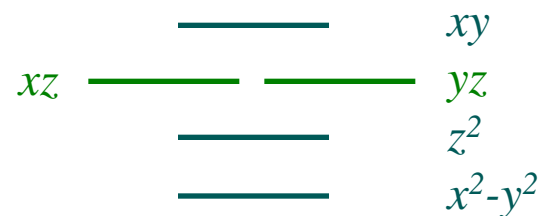
Chi-Cheng Lee, Wei-Guo Yin & Wei Ku

Phys. Rev. Lett. **103**, 267001 (2009)

Energy resolved, symmetry respecting Wannier function

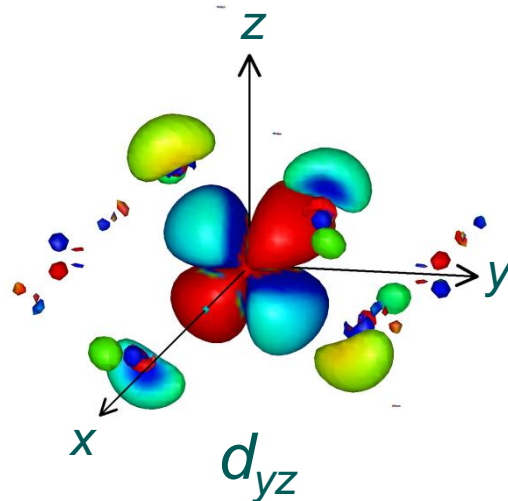
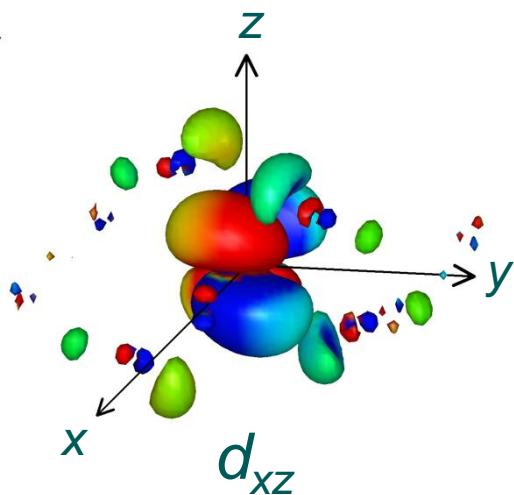


$$\begin{aligned}
 |\bar{R}n\rangle &= \sum_{\bar{k}m}^{(\text{energy window})} |\bar{k}m\rangle \langle \bar{k}m | \bar{R}n \rangle \\
 &= \frac{1}{\sqrt{N_{\text{cell}}}} \sum_{\bar{k}m} |\bar{k}m\rangle e^{-i\bar{k}\cdot\bar{R}} U_{mn}^{(\bar{k})} \\
 &= \frac{1}{\sqrt{N_{\text{cell}}}} \sum_{\bar{k}} \left(\sum_m U_{mn}^{(\bar{k})} |\bar{k}m\rangle \right) e^{-i\bar{k}\cdot\bar{R}}
 \end{aligned}$$



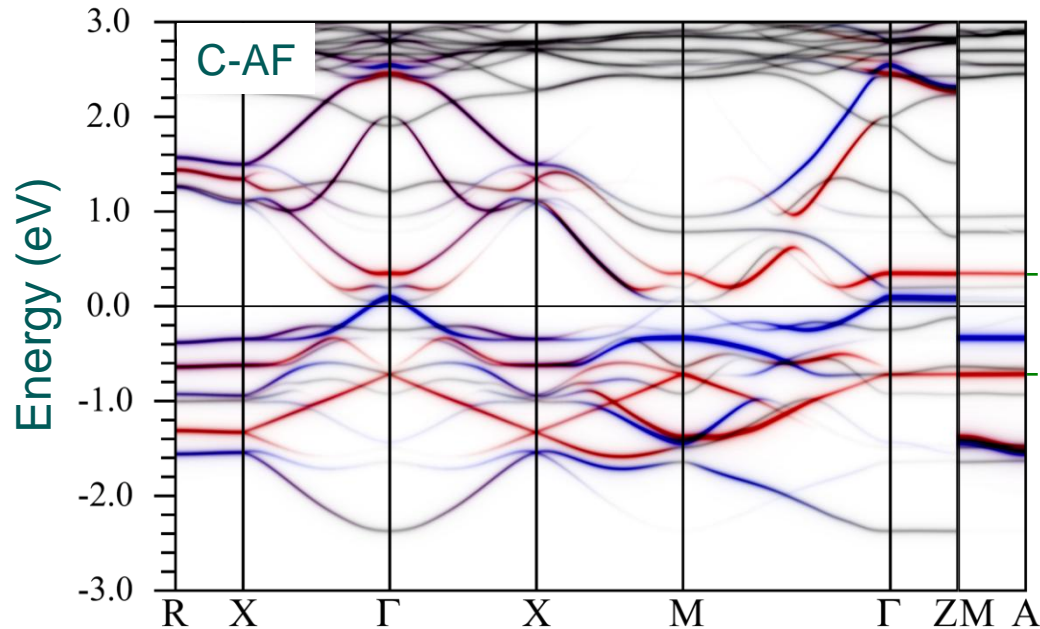
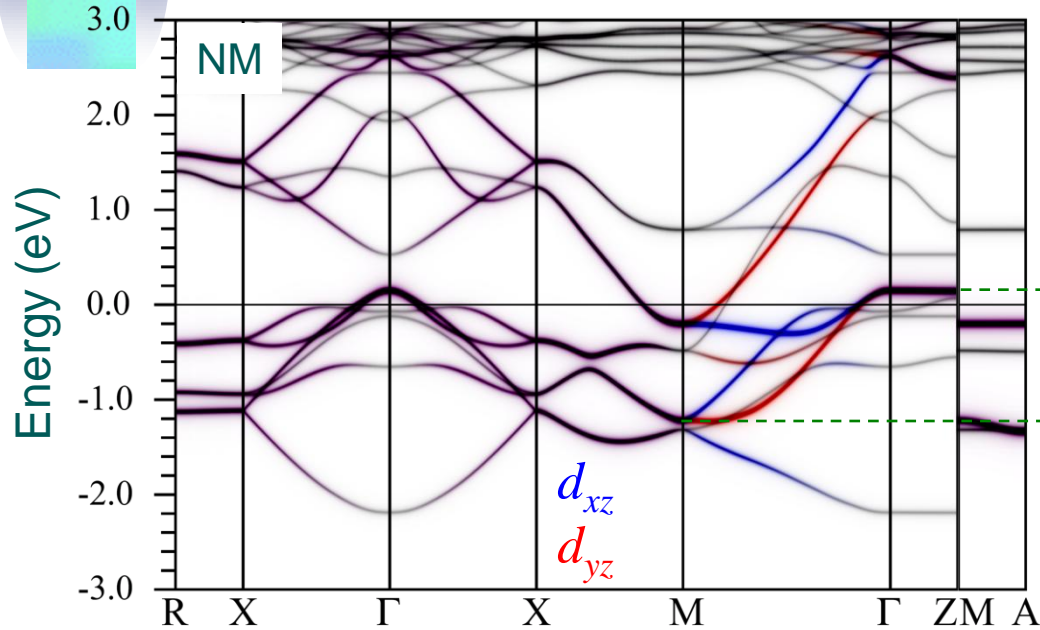
NM onsite energy (eV)

z^2	-0.03
x^2-y^2	-0.20
yz	0.10
xz	0.10
xy	0.34



- small crystal field splitting
- degenerate xz and yz
- orbital freedom !

Comparing LDA band structures



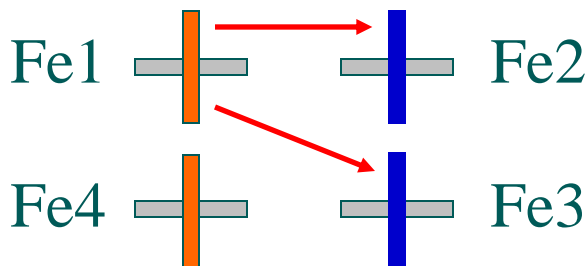
- in NM 1st-BZ
- d_{xz} & d_{yz} most relevant to the low-E
- Only d_{yz} splits strongly near E_F
- d_{yz} more spin polarized $\sim 0.34\mu_B$ than d_{xz} ($\sim 0.15\mu_B$)
- more different with $U=2eV$ 0.58 vs. $0.23\mu_B$
- orbital symmetry broken
- $\Delta \sim W$
- large (ω, \mathbf{k}) -space involved
- local picture more suitable
- Fermi surface nesting not essential
- SDW less convenient

unfolding methods see:

Wei Ku *et al.*, PRL **104**, 216401 (2010)

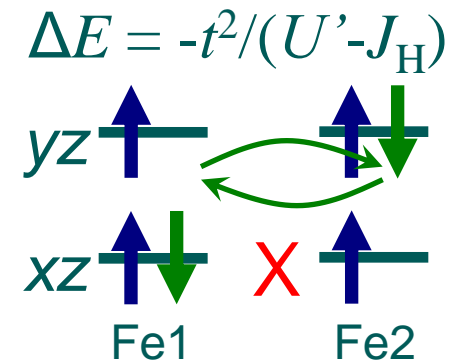
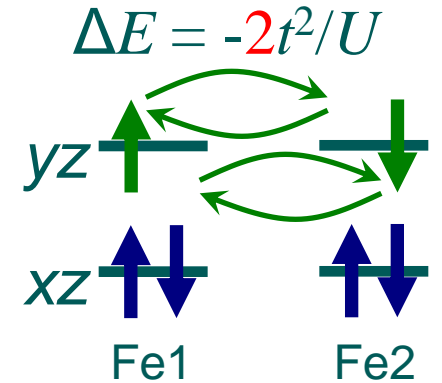
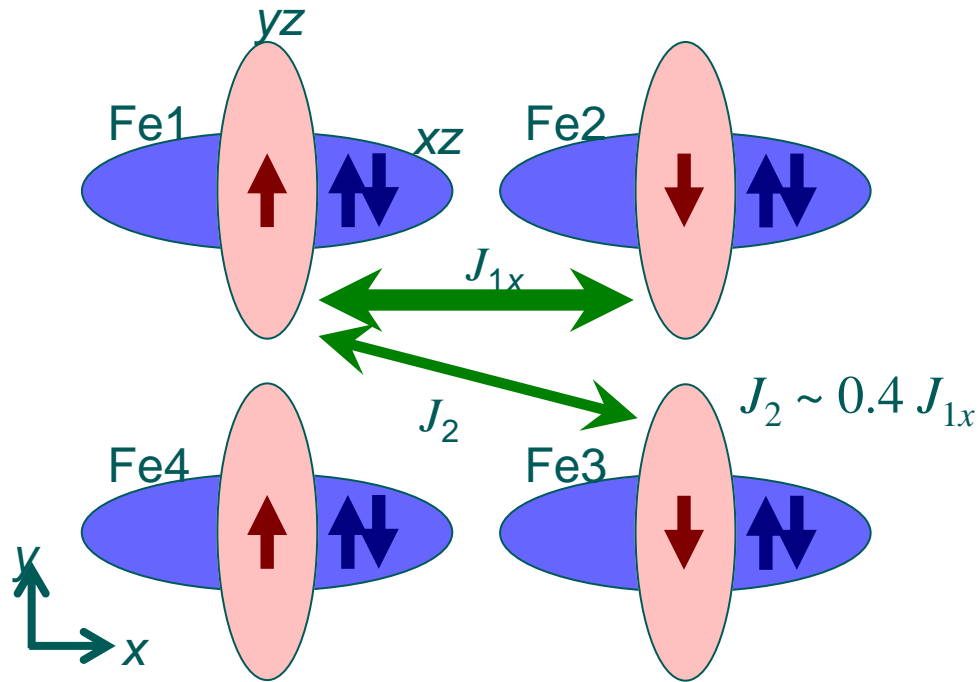
Anti-intuitive hopping parameters

$\langle \text{WFs} H \text{WFs} \rangle$	Fe1 z^2	x^2-y^2	yz	xz	xy
Fe2 (Fe4) z^2	0.13	0.31 (-0.31)	-0.10 (0.00)	0.00 (0.10)	0.00
x^2-y^2	0.31 (-0.31)	-0.32	0.42 (0.00)	0.00 (0.42)	0.00
yz	-0.10 (0.00)	0.42 (0.00)	-0.40 (-0.13)	0.00	0.00 (0.23)
xz	0.00 (0.10)	0.00 (0.42)	0.00	-0.13 (-0.40)	-0.23 (0.00)
xy	0.00	0.00	0.00 (0.23)	-0.23 (0.00)	-0.30
Fe3 z^2	0.06	0.00	-0.08	0.08	0.26
x^2-y^2	0.00	-0.10	0.12	0.12	0.00
yz	0.08	-0.12	0.25	-0.07	-0.05
xz	-0.08	-0.12	-0.07	0.25	0.05
xy	0.26	0.00	0.05	-0.05	0.16



- Unusual coupling direction
- Cubic symmetry **broken seriously** by As
→ **Fe-As phonon** modes important
- **Perpendicular** hopping direction!

C-AF magnetic structure and ferro-orbital order



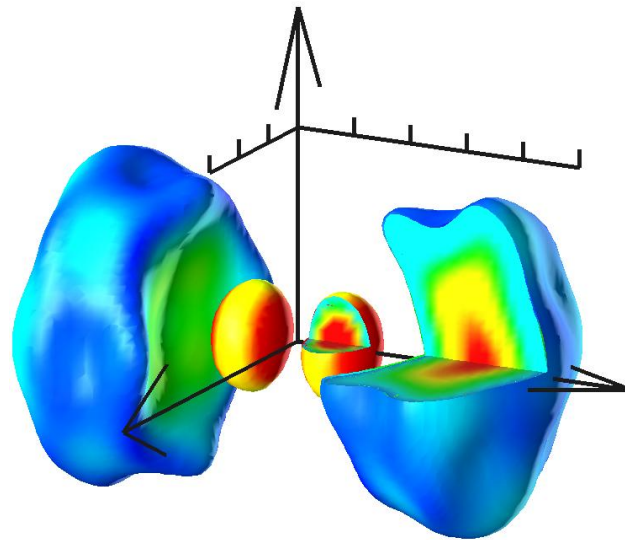
- Strongly anisotropic super-exchange: $J_{1x} > J_2 \gg J_{1y}$
 - no competition with G-AF at all! $J_1 \sim 2J_2$ irrelevant!
 - single orbital Heisenberg model inadequate
- Orbital polarization and ferro-orbital correlation important
 - rotational symmetry breaking
- Unusual coupling direction and strong anisotropic hoppings!
 - common staggered OO loses
 - $a > b$: AF across long bond (rare)



Outlines

- Simple illustrations:
 - Orbital order in LaMnO_3
 - Gapless CDW in TaSe_2
 - Ferro-orbital order in Fe-SCs
- ➔ ● Interacting problems:
 - Local excitations
 - Propagation and decay of local excitations
- Effects of disordered impurities in Fe-SCs:
 - Substitution of Fe: doping or not?
 - Fe vacancy: “violation” of Luttinger theorem
 - Ru substitution: realization of superdiffusion

Propagation of local excitations in strongly correlated materials



P. Abbamonte *et al.*, PNAS **105**, 12159 (2008)

C-C Lee *et al.*, Phys. Rev. Lett. **111**, 157401 (2013)



Acknowledgement

Funding sources

- Basic Energy Science, Department of Energy
- DOE-CMSN
- Taiwanese Student Fellowship

Theoretical works

- Chi-Cheng Lee (BNL & Tamkang Univ.)
- Chen-Lin Yeh (BNL & Tamkang Univ.)
- Hung-Chung Hsueh (Tamkang Univ.)

Experiments

- Peter Abbamonte, Xiaoqian M. Chen, Yu Gan (UIUC)
- Ben C. Larson (ORNL)
- John Hill (BNL), Young Cai (BNL)
- Andrivo Rusydi (NUS, Singapore)
- Young-June Kim (U. Toronto, Canada)
- Nozomu Hiraoka (Sping-8, Japna)



Chi-Cheng Lee
Sinica

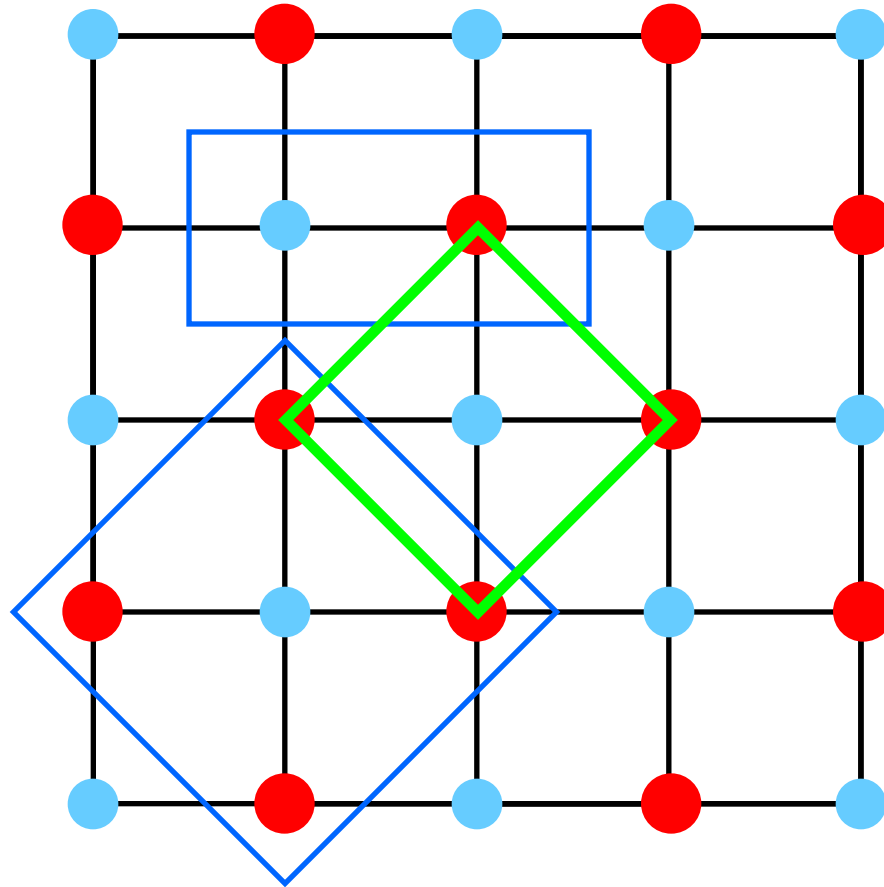


Local Picture for Strongly Correlated Systems

$$H = H_0 + V = \boxed{H_{local}} + H_{nonlocal}$$

- V too big for perturbation
- Maximize the terms in the “local” part
 - symmetric Wannier Representation → defines “local”
- Treat local part “accurately”
- Add non-local part as modification

How to define “local” in CT-insulators?



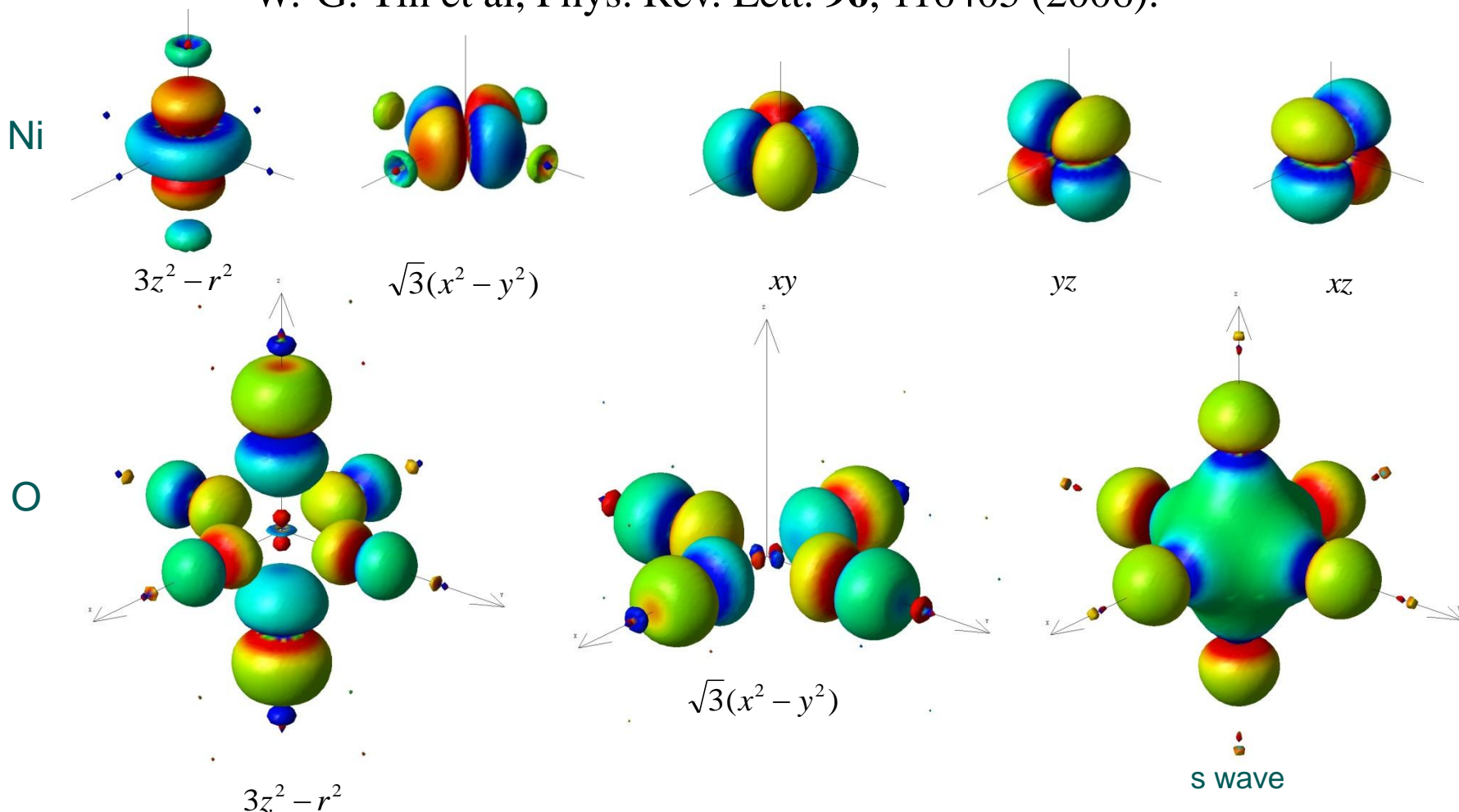
- Periodic symmetry
- Point group symmetry
- Simultaneously keep both? How to split the Hilbert space?

Symmetric Wannier Functions for CT-Insulators

W. Ku et al., Phys. Rev. Lett. **89**, 167204 (2002).

R. L. Barnett et al., Phys. Rev. Lett. **96**, 026406 (2006).

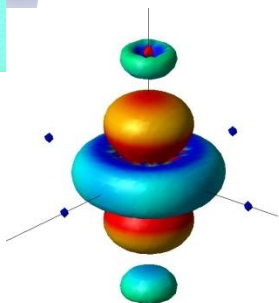
W.-G. Yin et al, Phys. Rev. Lett. **96**, 116405 (2006).



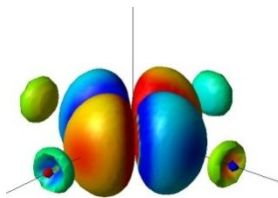
- O-*p* orbitals → additional Ni-*d* orbitals (no double counting of O orbitals)
- “local” is now defined by this “super-atom”

Super Atom for Charge Transfer Insulator

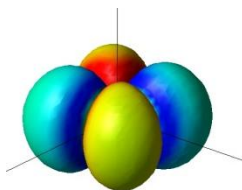
Ni



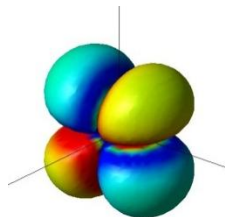
$$3z^2 - r^2$$



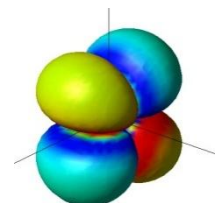
$$\sqrt{3}(x^2 - y^2)$$



$$xy$$

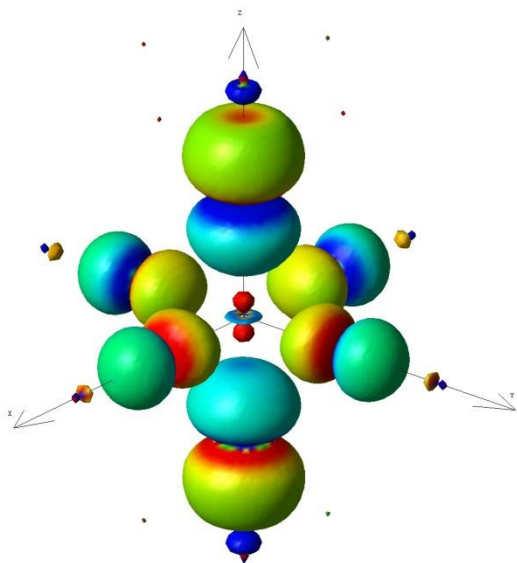


$$yz$$

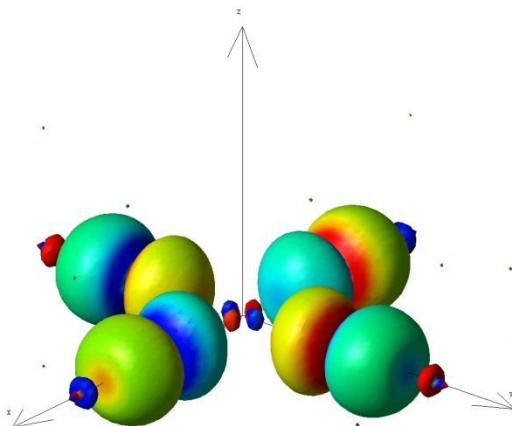


$$xz$$

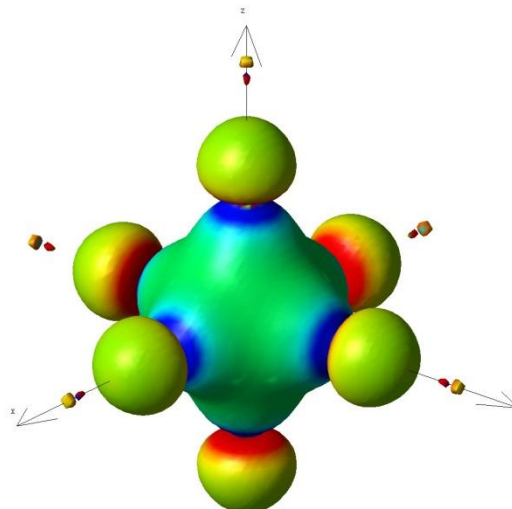
O



$$3z^2 - r^2$$



$$\sqrt{3}(x^2 - y^2)$$



s wave

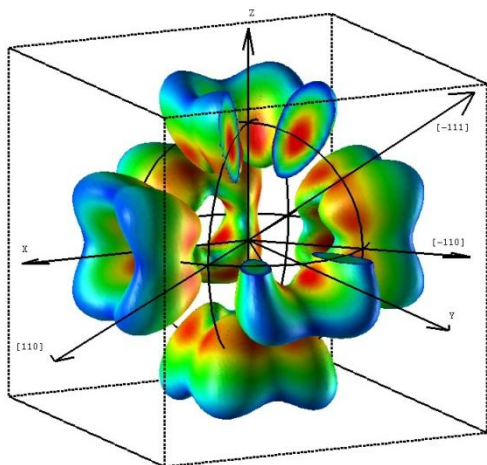
$$H = H_{local} + H_{nonlocal}$$

(exact) (modification)

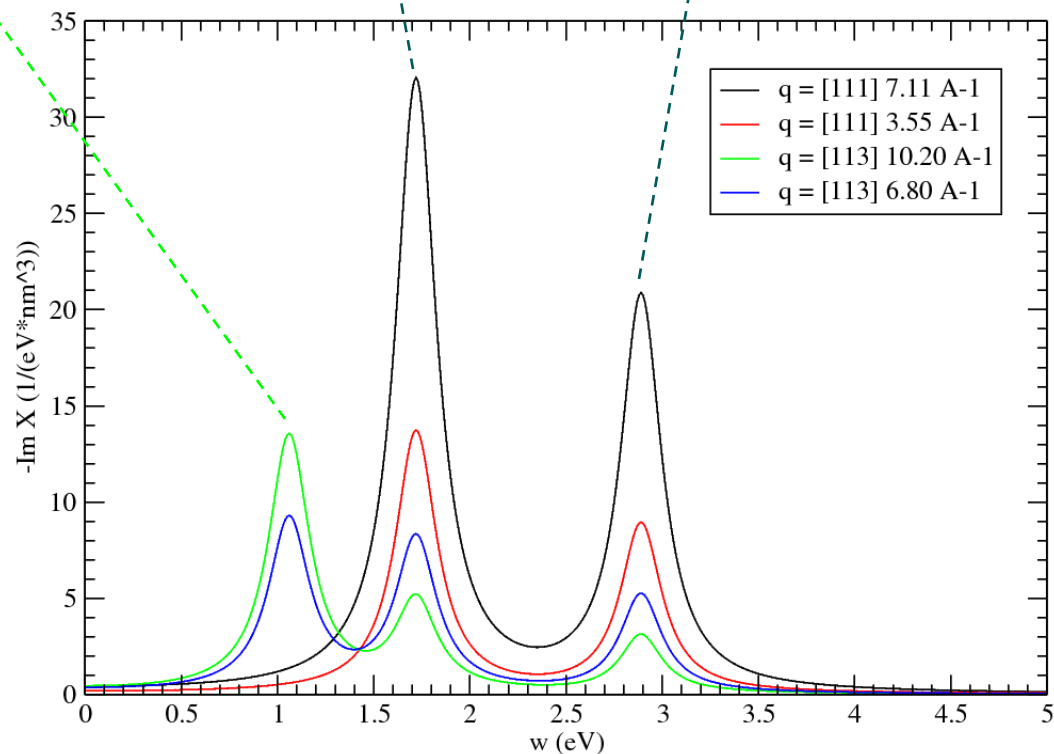
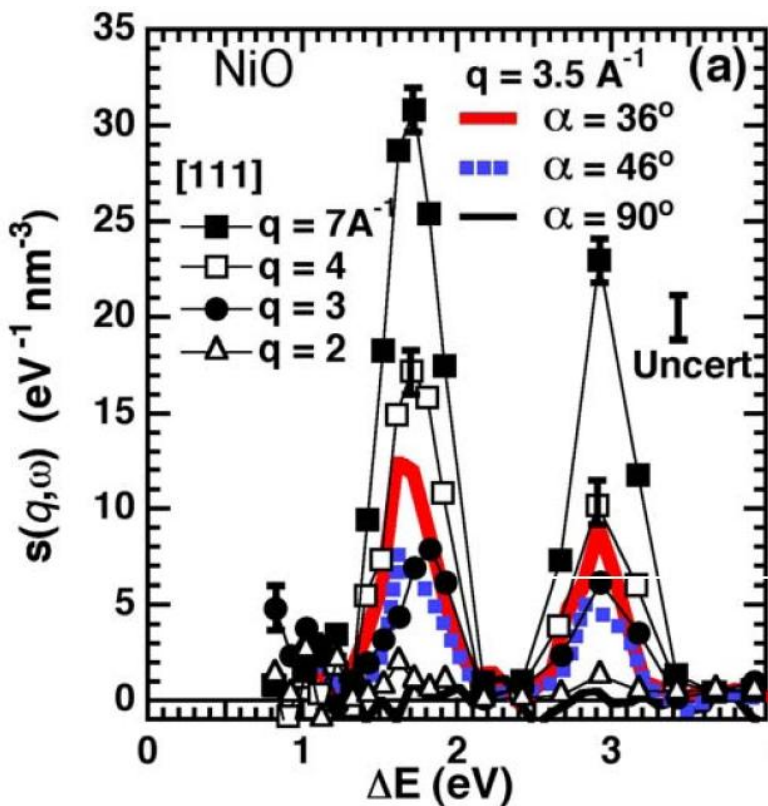
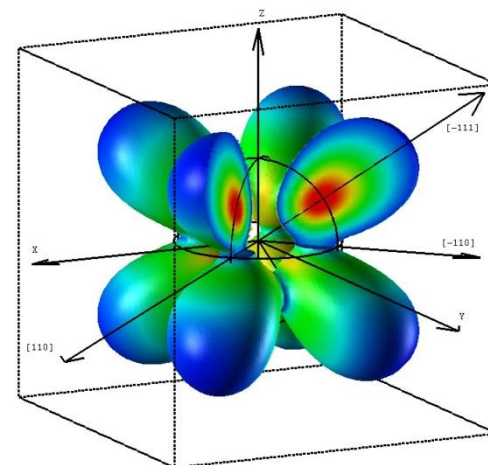
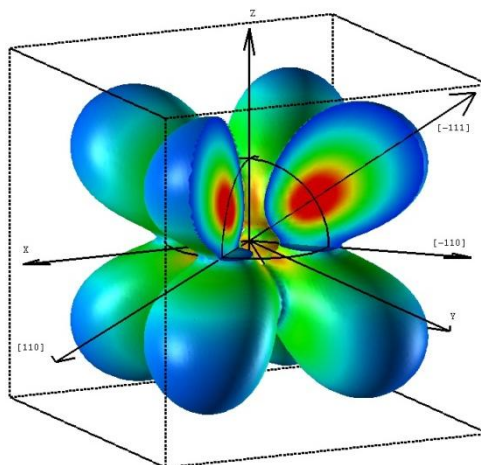
Maximize the contributions of "local atom"

Density response for super atom

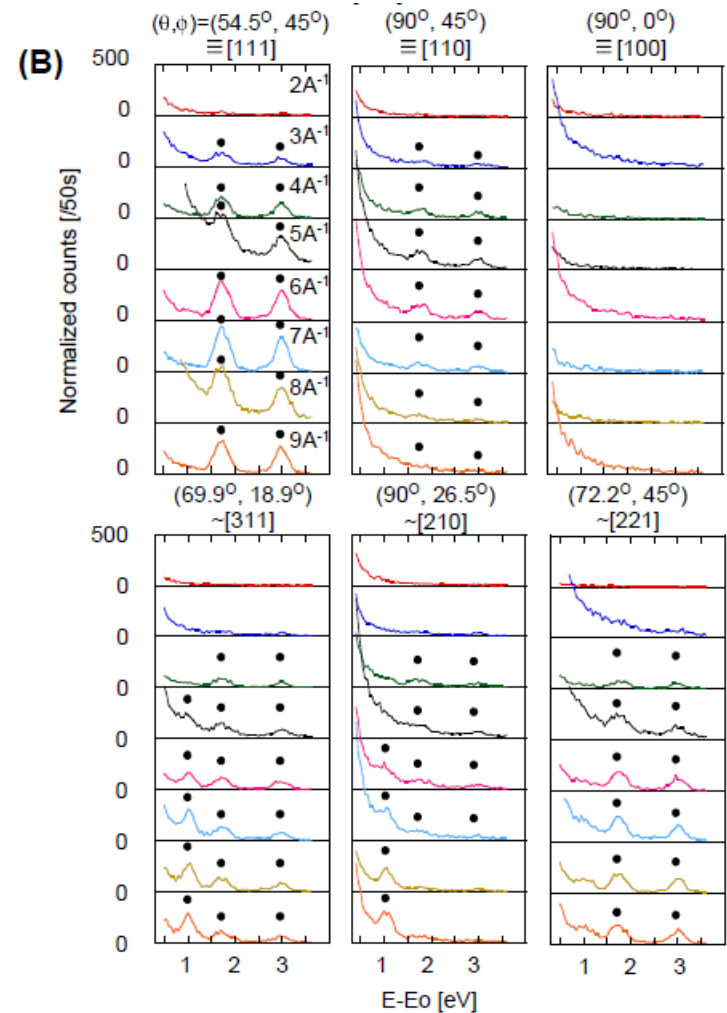
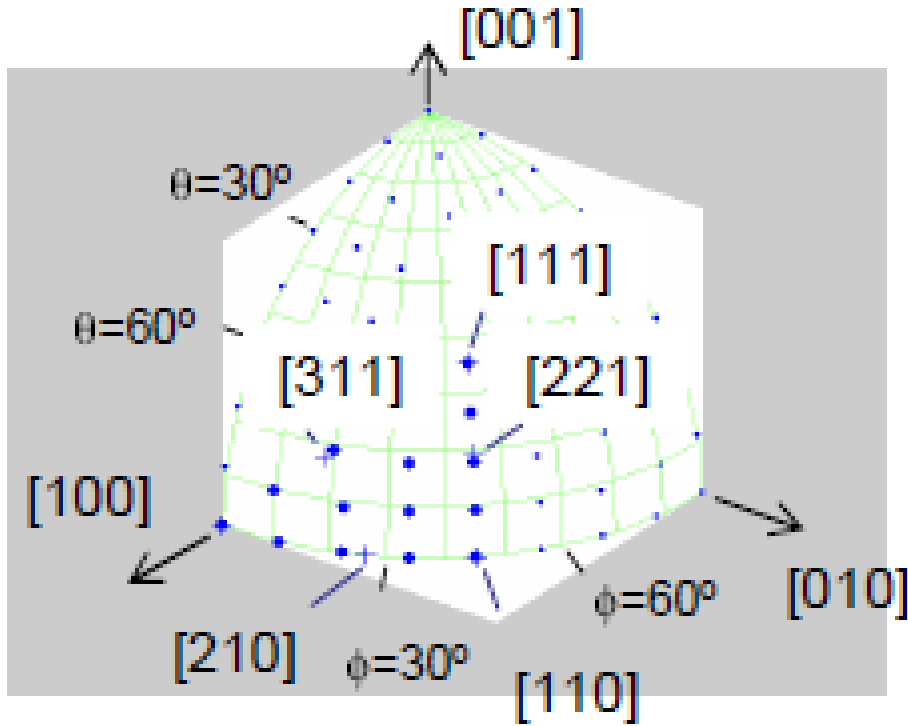
(eg-t2g=0.65eV)



(antibonding-type)

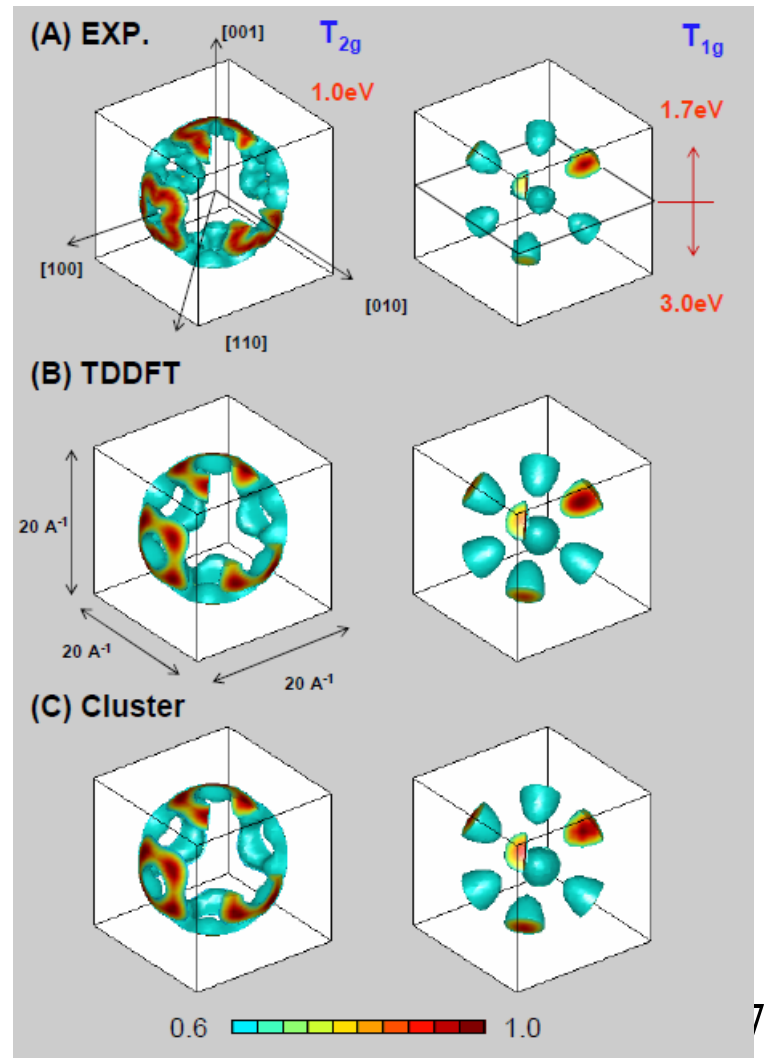
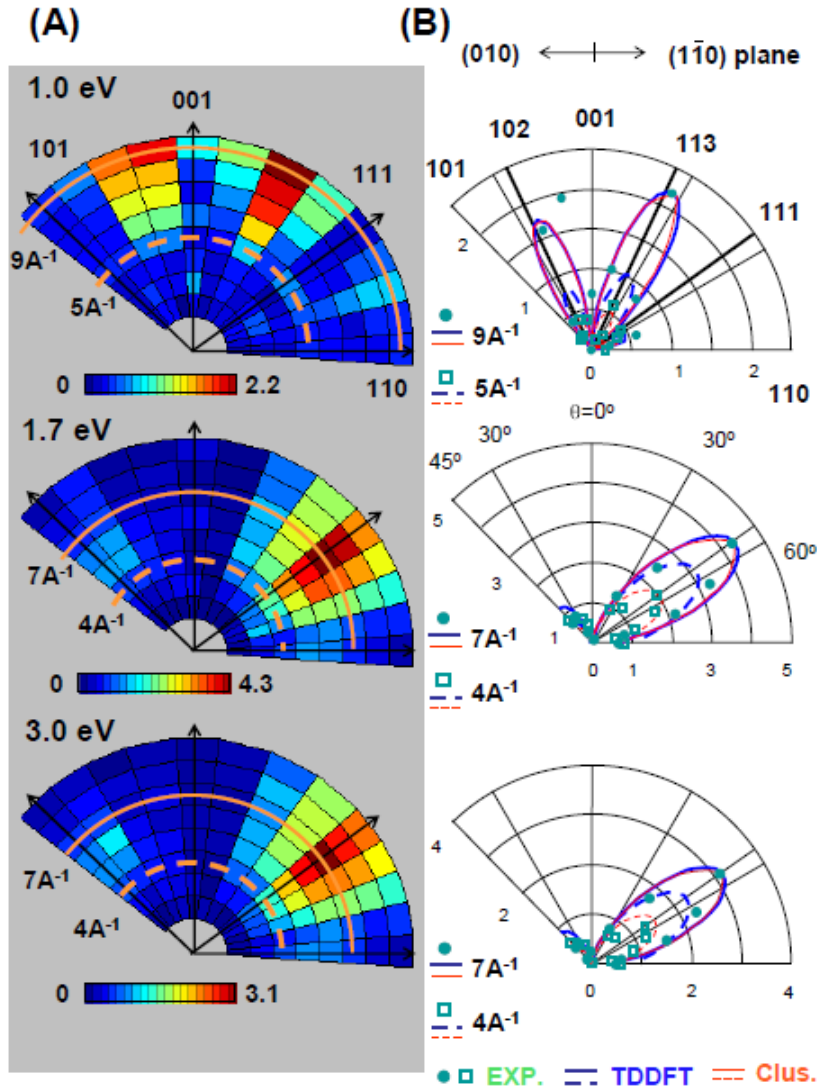


3D Map of Local Excitations in the Mott Gap in NiO



- 1eV excitation found !
- very different q-dependence of spectral weight

3D Map of Local Excitations in the Mott Gap in NiO



- Detailed comparison: exp vs. TD-LDA+ U & cluster model
 - good general agreement with some discrepancies in details



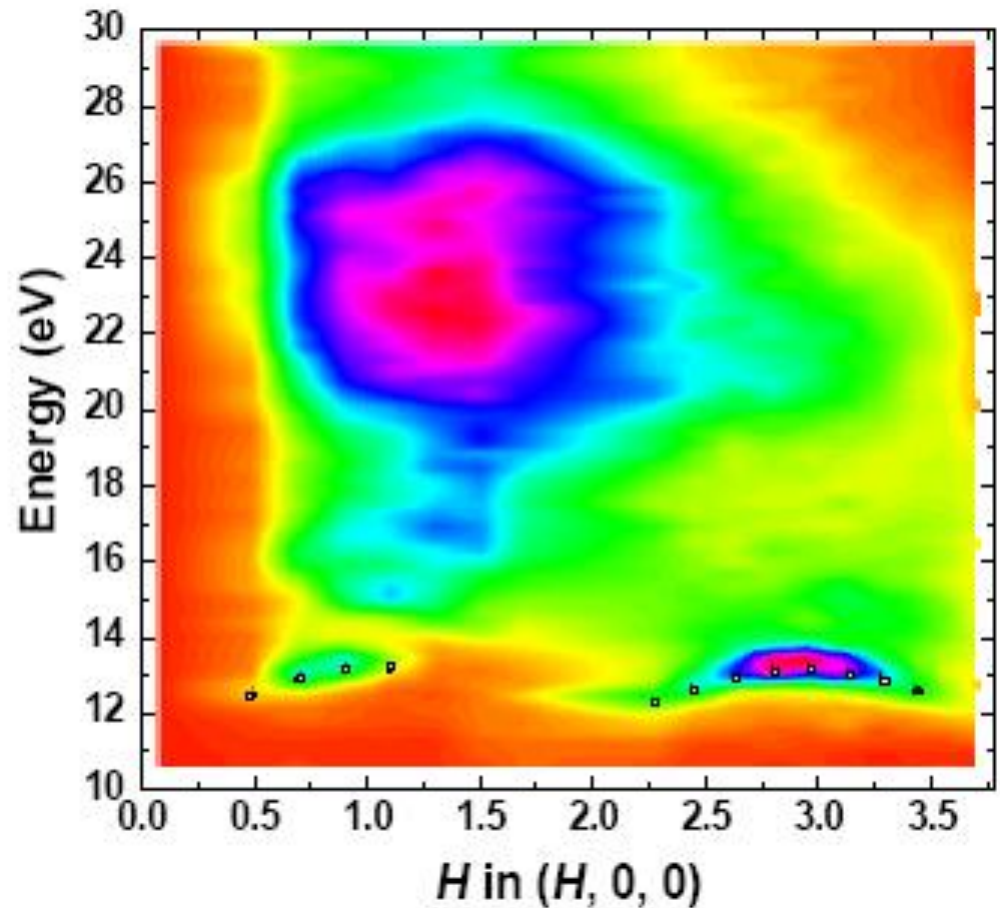
How does the local excitations propagate and decay?

Propagation of Tightly-Bound Excitons: case study of LiF

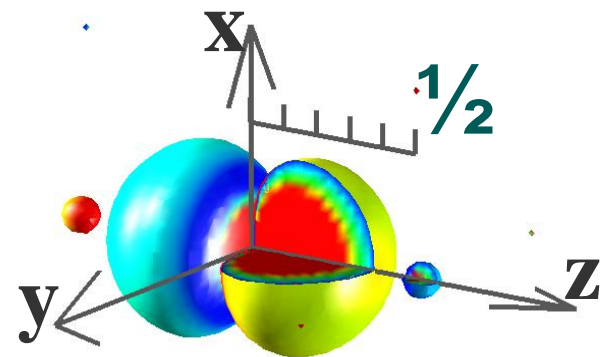
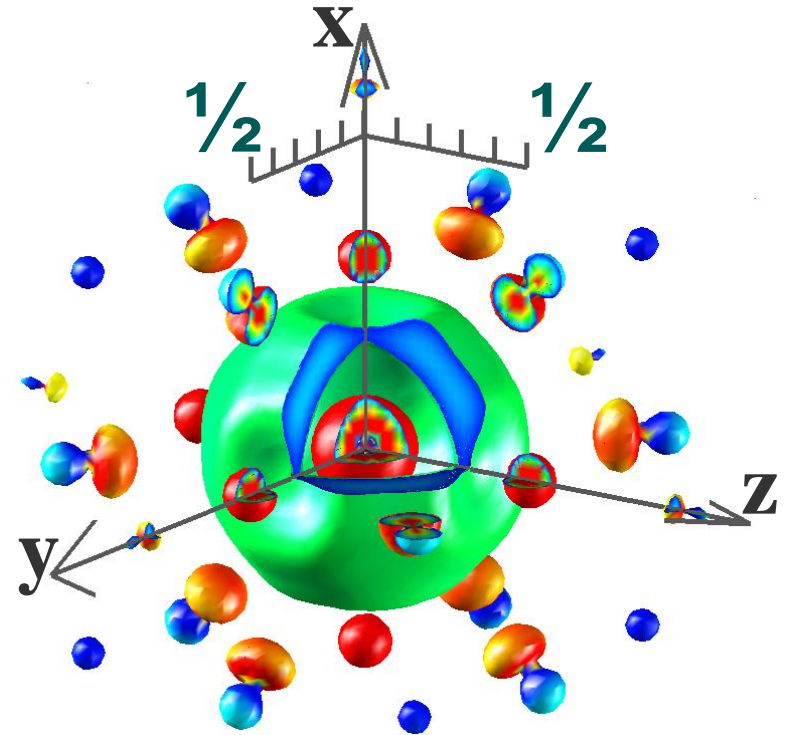
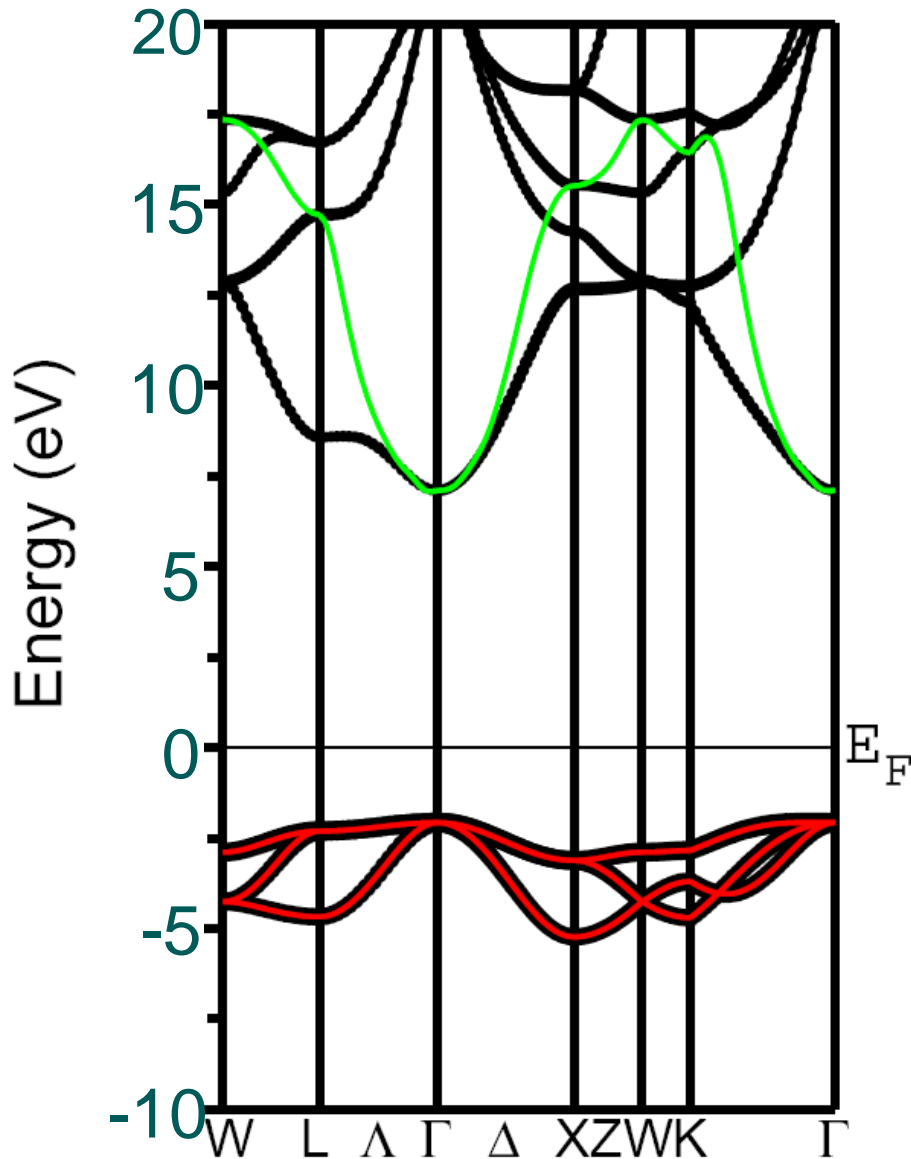
- Tightly bound exciton
- Charge transfer insulator
→ p-h in different atoms
- Frenkel or Wannier exciton ?
- Dispersion
→ propagation in space/time

Inelastic X-ray scattering

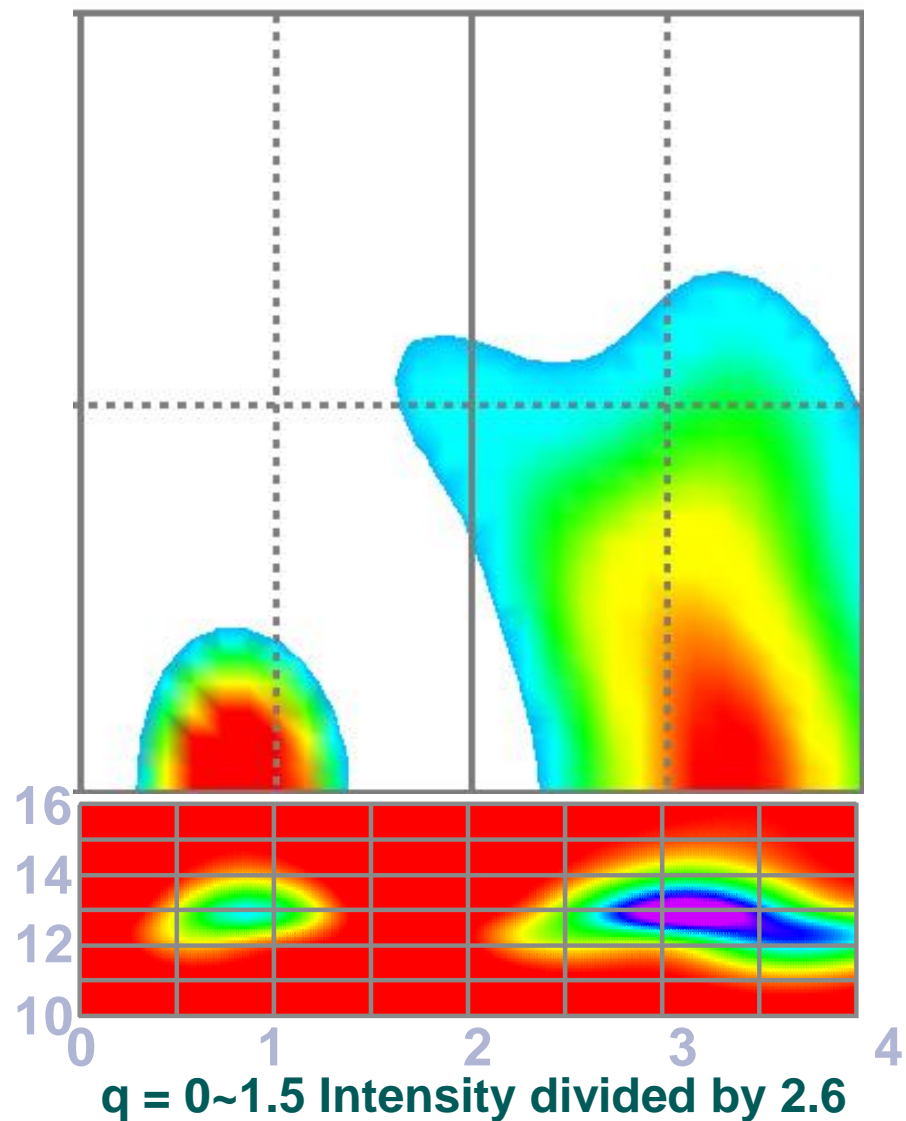
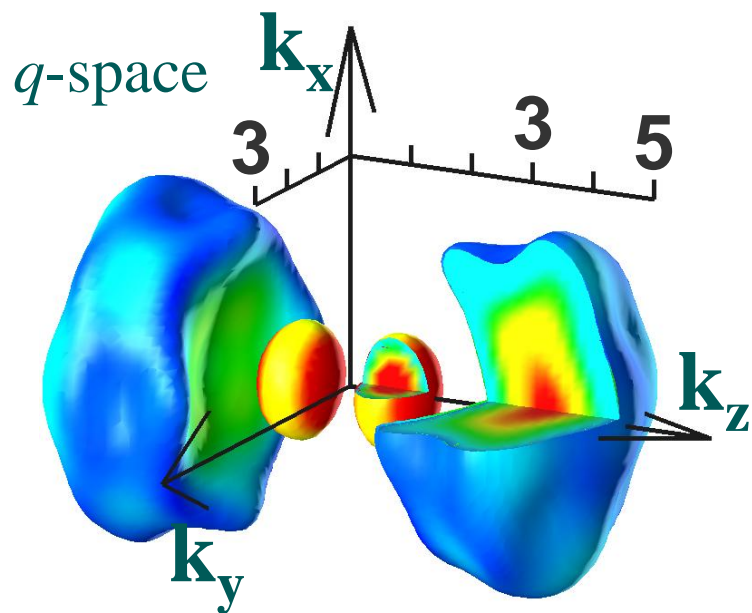
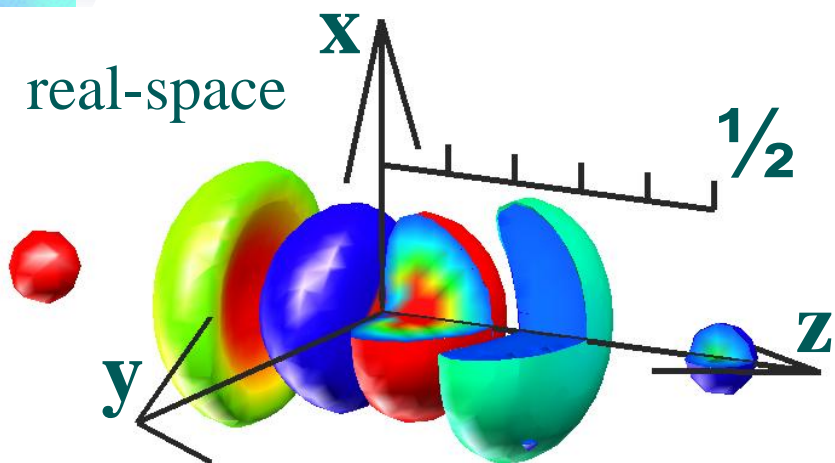
- Structured spectral weight
- Clear dispersion at large q !
- observe fs dynamics



Excitons in LiF as a Frenkel Exciton in a “Super Atom”



Matrix Element and Structure in q -space





Propagation of tightly bound excitons

- Treat tightly bound excitons (and other local excitations) as a composite boson. Define its propagation kinetic kernel T via local and full propagator $D[H_L]$ and $D[H]$:

$$D[H] = D[H_L] + D[H_L]TD[H]$$

- T integrates out all the pair fluctuation in space and encapsulates propagation and decay processes.
- We then approximate T using unbound exciton propagator

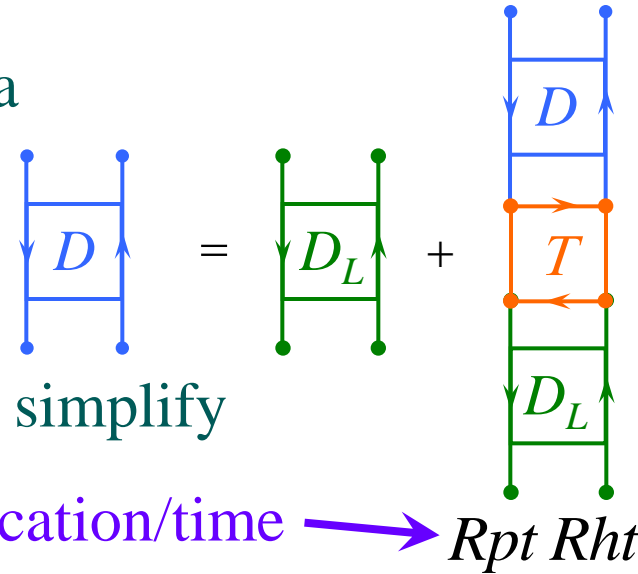
$$T = D^{-1}[H_L] - D^{-1}[H] \sim D_0^{-1}[G[H_L]] - D_0^{-1}[G[H]]$$

- Separation of local many-body problem from non-local propagation
- Many orders of magnitudes cheaper than Bethe-Salpeter equation

Effective Two-Particle Hopping

Define effective two particle kinetic kernel T via

$$D[H] = D[H_L] + D[H_L]TD[H]$$



in the basis of local bound pair $b_{RN}^+ \equiv c_{Rp}^+ c_{Rh}$ and simplify

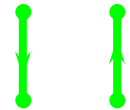
$$T = D^{-1}[H_L] - D^{-1}[H]$$

$$= \left(D_0^{-1}[G[H_L]] - I[H_L] \right) - \left(D_0^{-1}[G[H]] - I[H] \right)$$

$$\simeq D_0^{-1}[G[H_L]] - D_0^{-1}[G[H]]$$

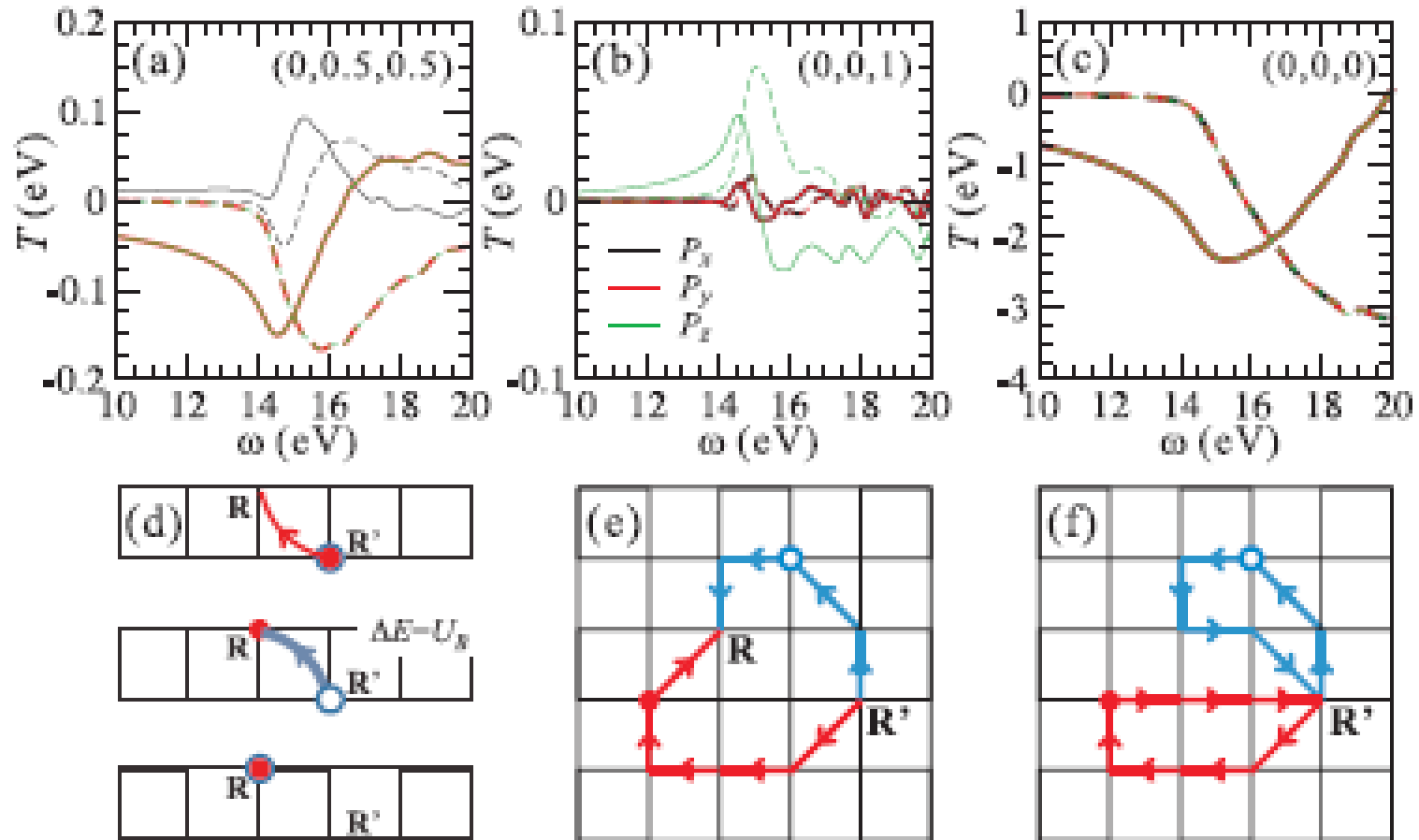
using the empty bubble

$$D_0(RN, R'N'; t, t') = G(Rp, R'p'; t, t') G(R'h', Rh; t', t)$$



T gives hopping of p-h pair in real space \rightarrow dispersion in q -space

Effective Two-Particle Hopping in LiF



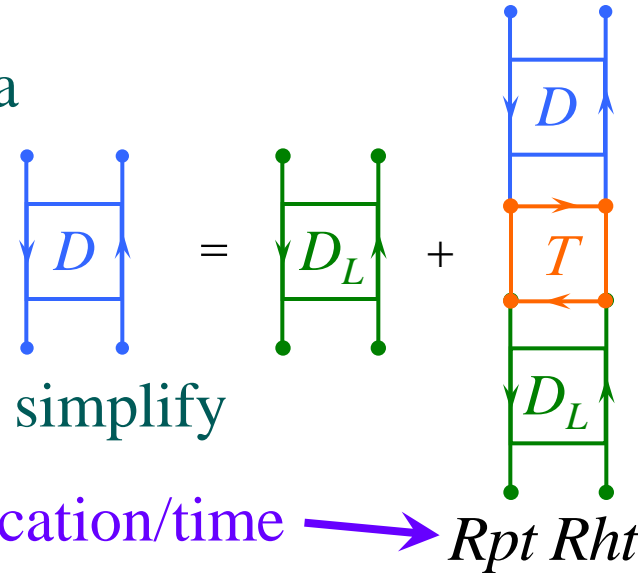
$T(\omega)$ is **complex** and strongly ω -dependent to fully account for

1. Landau continuum (integrating out virtual pair breaking processes)
 \rightarrow exact for $E_b = 0$
2. Lower mobility with stronger p-h binding \rightarrow correct t^2/E_b behavior
3. Renormalization of on-site energy from **kinetic** energy

Effective Two-Particle Hopping

Define effective two particle kinetic kernel T via

$$D[H] = D[H_L] + D[H_L]TD[H]$$



in the basis of local bound pair $b_{RN}^+ \equiv c_{Rp}^+ c_{Rh}$ and simplify

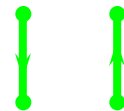
$$T = D^{-1}[H_L] - D^{-1}[H]$$

$$= \left(D_0^{-1}[G[H_L]] - I[H_L] \right) - \left(D_0^{-1}[G[H]] - I[H] \right)$$

$$\simeq D_0^{-1}[G[H_L]] - D_0^{-1}[G[H]]$$

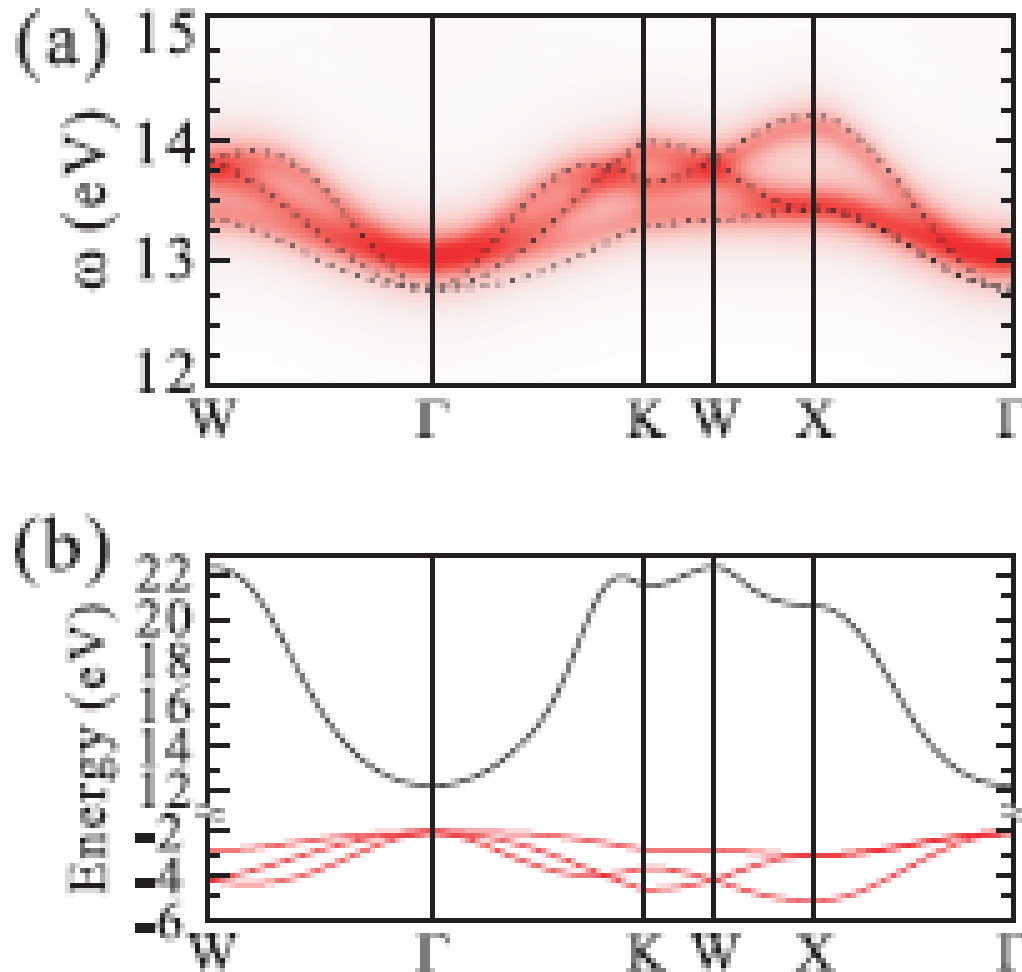
using the empty bubble

$$D_0(RN, R'N'; t, t') = G(Rp, R'p'; t, t') G(R'h', Rh; t', t)$$



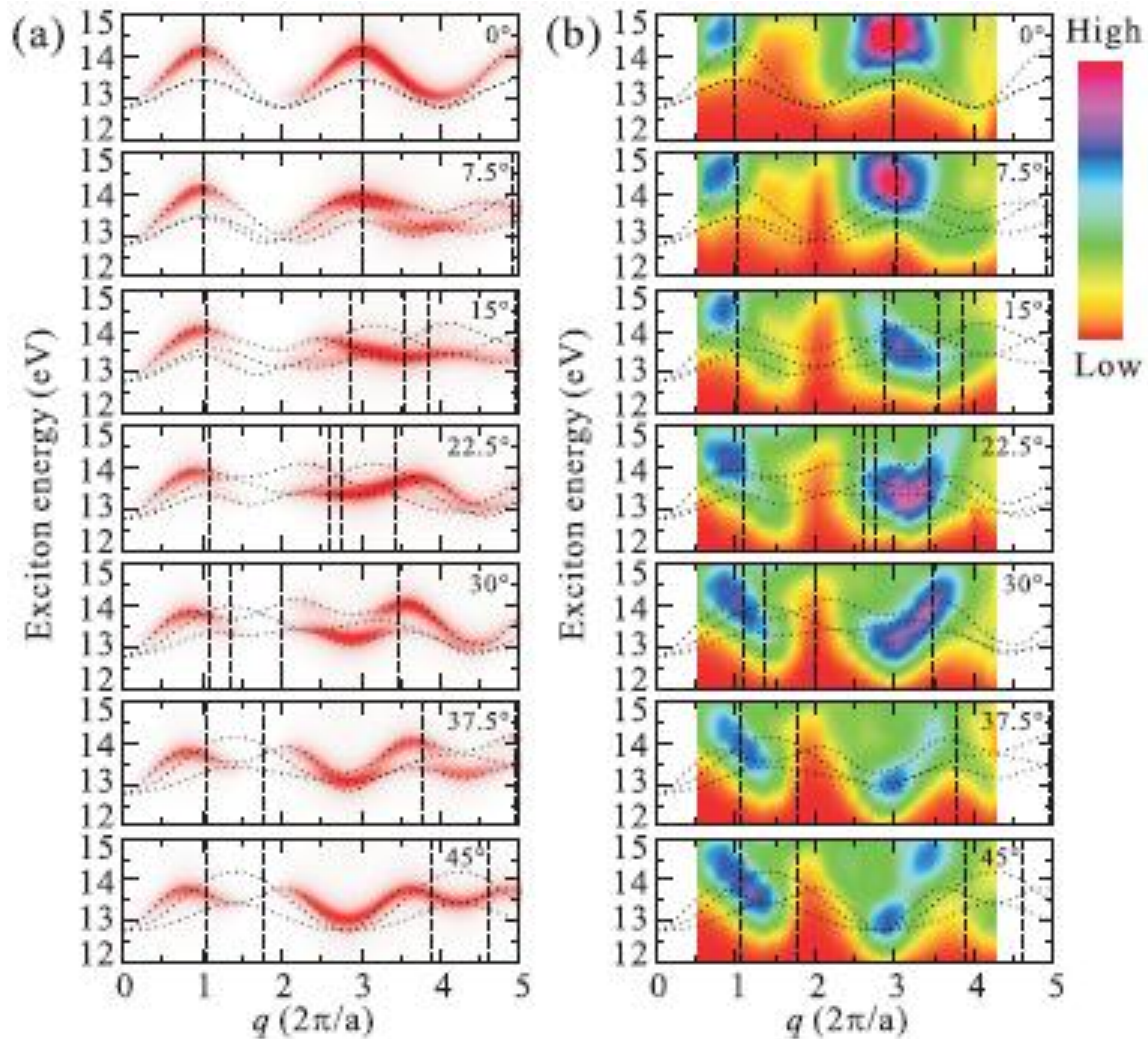
T gives hopping of p-h pair in real space \rightarrow dispersion in q -space

Exciton Band Structure



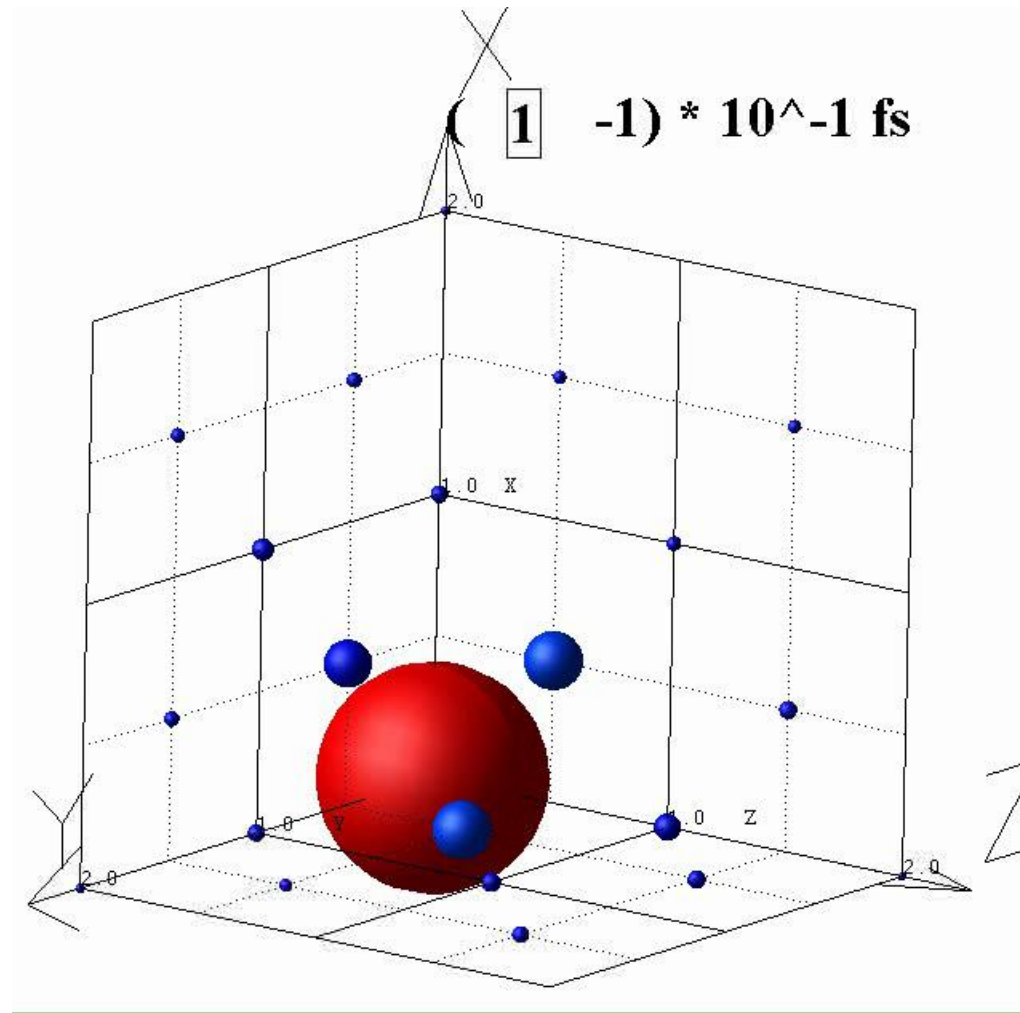
1. Full results similar to the diagonalization of $\omega_{exciton} + \text{Re}T(\tilde{\omega}_{exciton})$
2. Similar dispersion to the F p -bands (same symmetry)

Observation of Multiple Exciton Bands



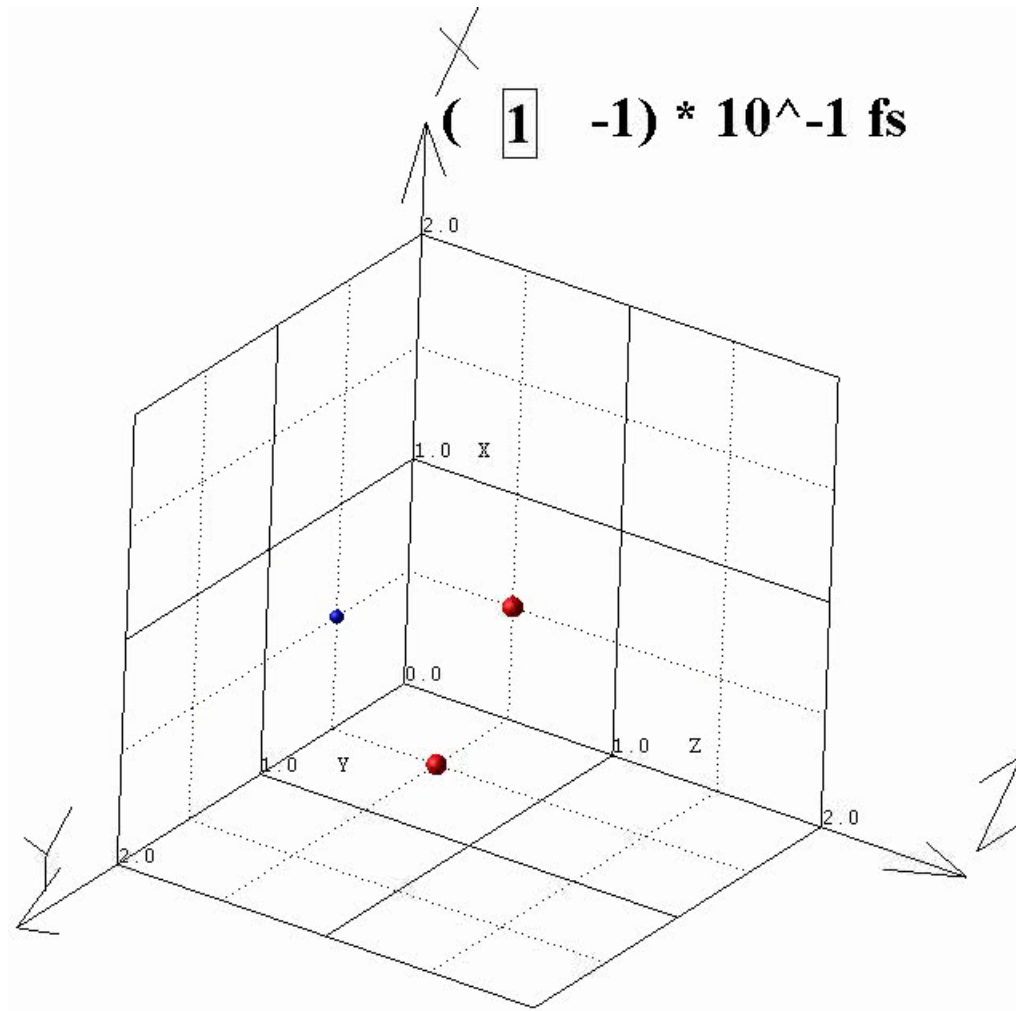
1. Similar weight in momentum space
2. Similar dispersion
3. Switching of bands (breaks in intensity)

Time Evaluation of Charge Fluctuation in LiF



- exp. 2D map of exciton propagation is in preparation

Time Evaluation of Charge Fluctuation in LiF



- exp. 2D map of exciton propagation is in preparation



Outlines

- Simple illustrations:
 - Orbital order in LaMnO_3
 - Gapless CDW in TaSe_2
 - Ferro-orbital order in Fe-SCs
- Interacting problems:
 - Local excitations
 - Propagation and decay of local excitations
- ➔ ● Effects of disordered impurities in Fe-SCs:
 - Substitution of Fe: doping or not?
 - Fe vacancy: “violation” of Luttinger theorem
 - Ru substitution: realization of superdiffusion



Treating materials with disordered impurities

T. Berlijn, D. Volja, and Wei Ku, PRL **106**, 077005 (2011)

For various applications, see

T.S. Herng, *et al.*, *Phys. Rev. Lett.* **105**, 207201 (2010)

Tom Berlijn, *et al.*, *Phys. Rev. Lett.* **108**, 207003 (2012)

Tom Berlijn, *et al.*, *Phys. Rev. Lett.* **109**, 147003 (2012)

L.-M. Wang, *et al.*, *Phys. Rev. Lett.* **110**, 037001 (2013)



Acknowledgement

Funding sources

Basic Energy Science, Office of Science, Department of Energy



Dmitri Volja
MIT



Tom Berlijn
BNL



Limin Wang
BNL

Collaborators

P. Hirschfeld, & H.-P. Chen, U of Florida



First-principles methods for disorder

Relevance

- Defects, vacancy, chemical substitution, doping & intercalation
- Thermal/quantum fluctuation induced spatial inhomogeneity
- **Paramagnetic** phase with disordered local moment

Different approaches

- Mean-field potential: virtual crystal approximation
- Mean-field scattering: coherent potential approximation
- Perturbation: diagrammatic summation
- Non-perturbed direct method: configuration average over **super cells**

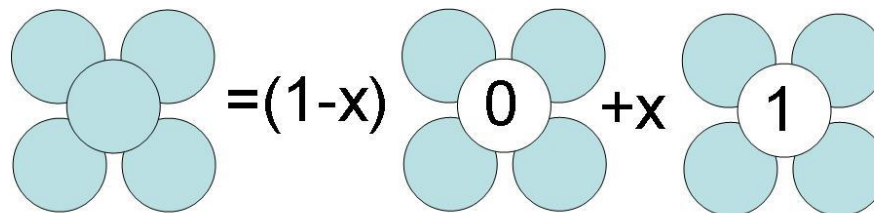
Mean-field approaches

VCA: Virtual Crystal Approximation

$$V_{\text{virtual crystal}} = (1-x) V_0 + x V_1$$

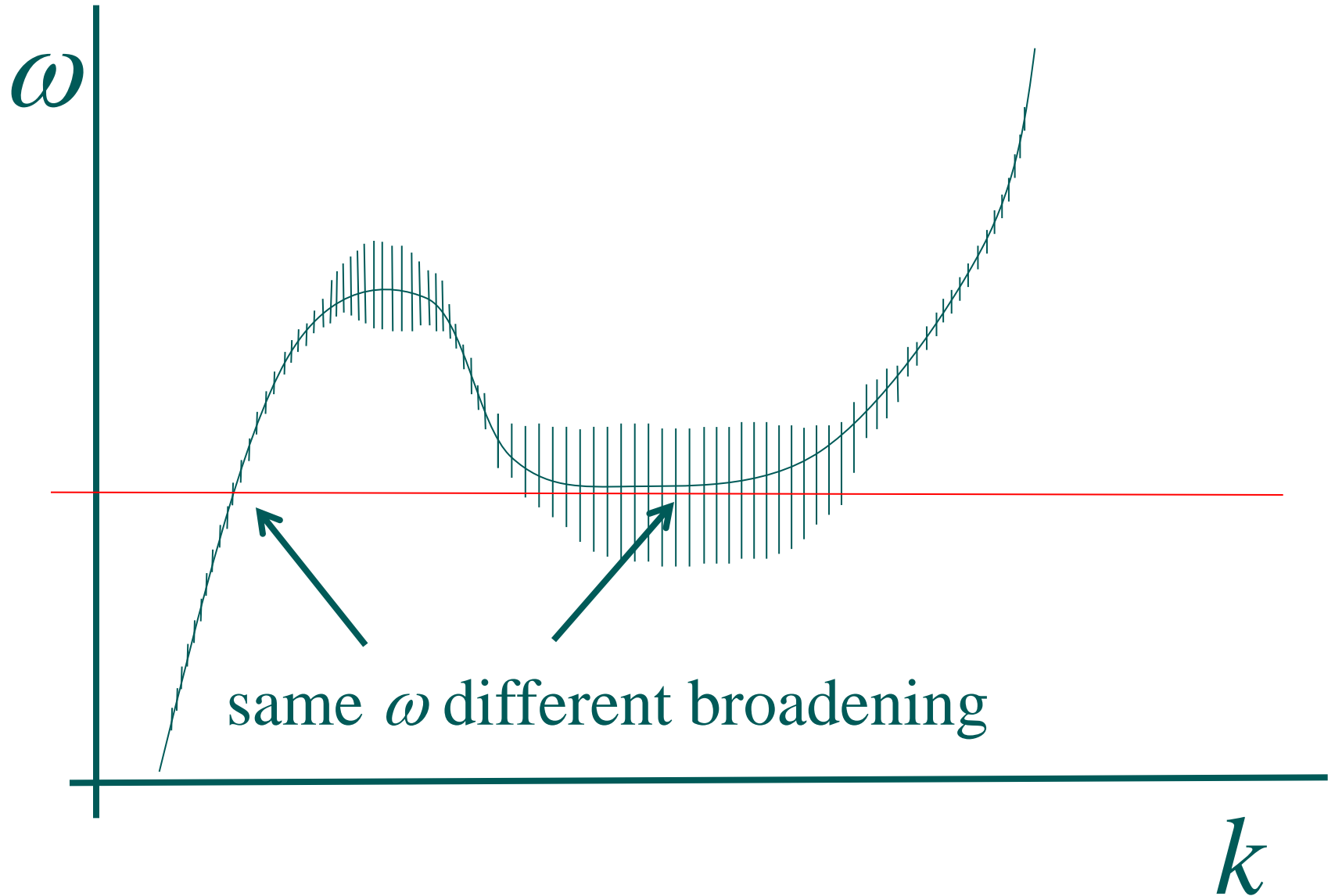
no scattering

CPA: Coherent Potential Approximation



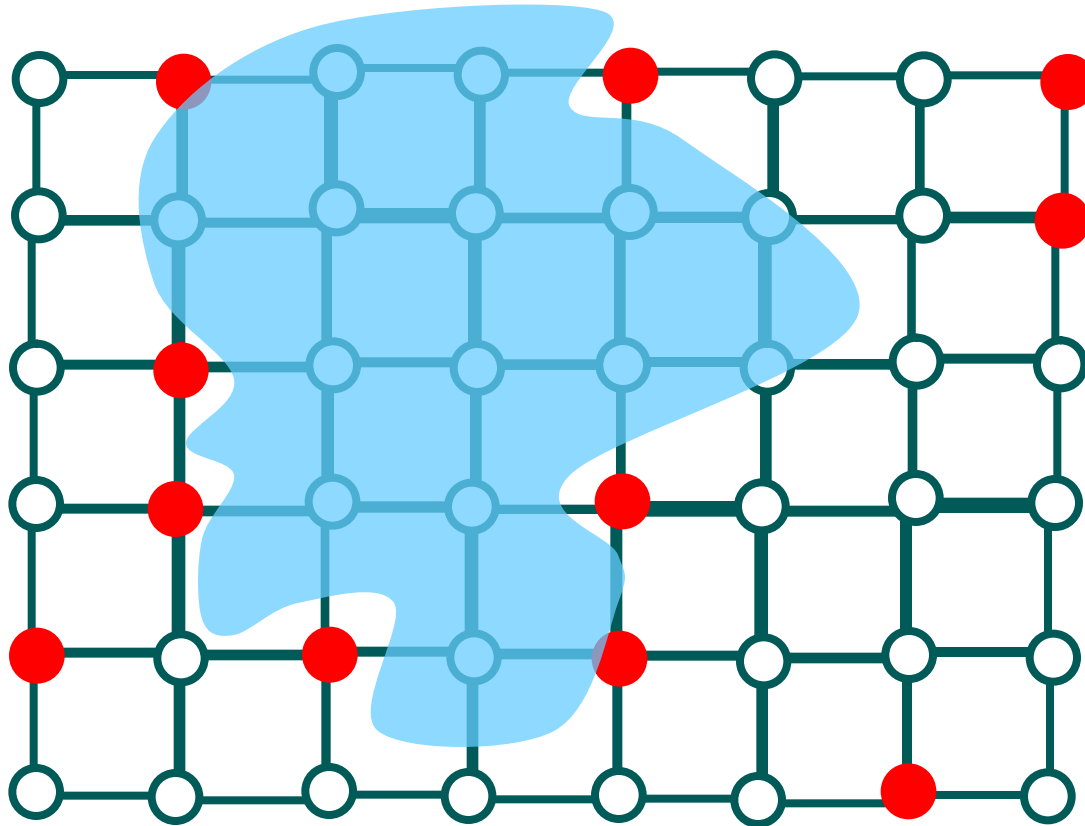
lack cluster-scattering

Non-local physics: k -dependent broadening



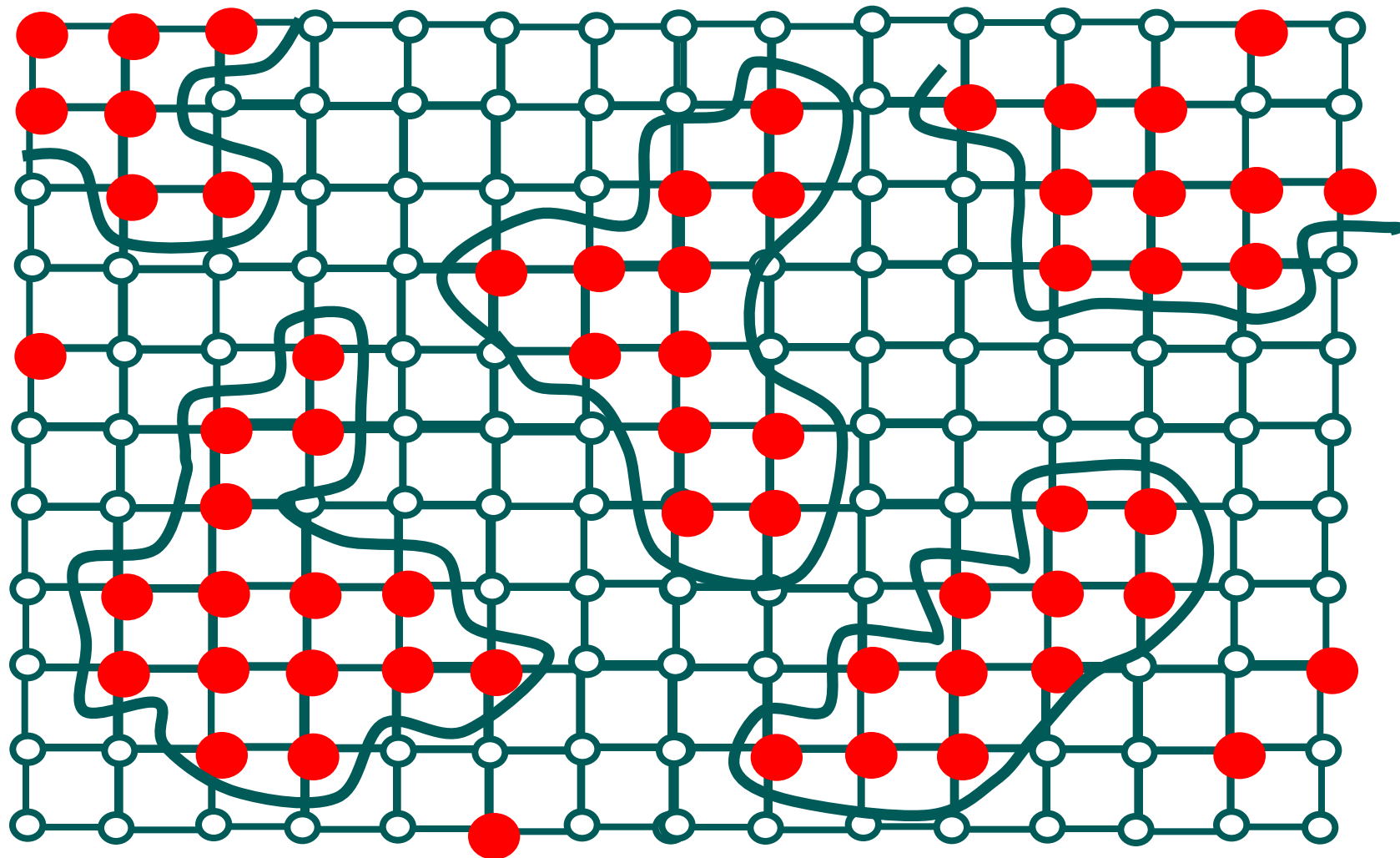


Non-local physics: large-sized impurity states



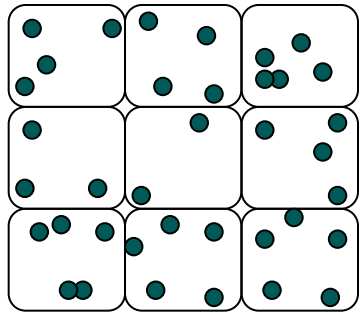
Anderson localization, multiple scattering required

Non-local physics: short-range order



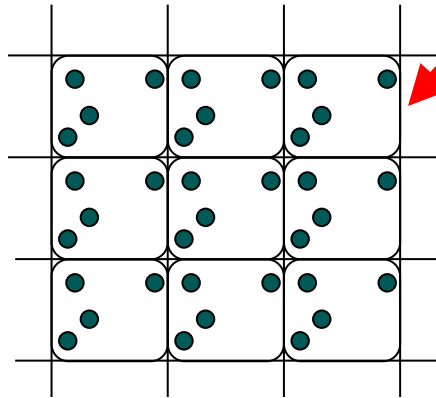
Configuration average over super cell

disordered system

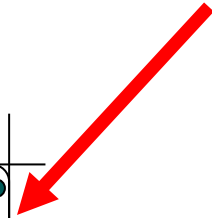


\approx

configuration 1

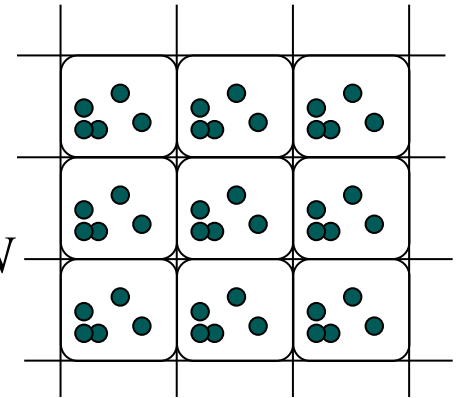


big super cells



+ ... +

configuration N

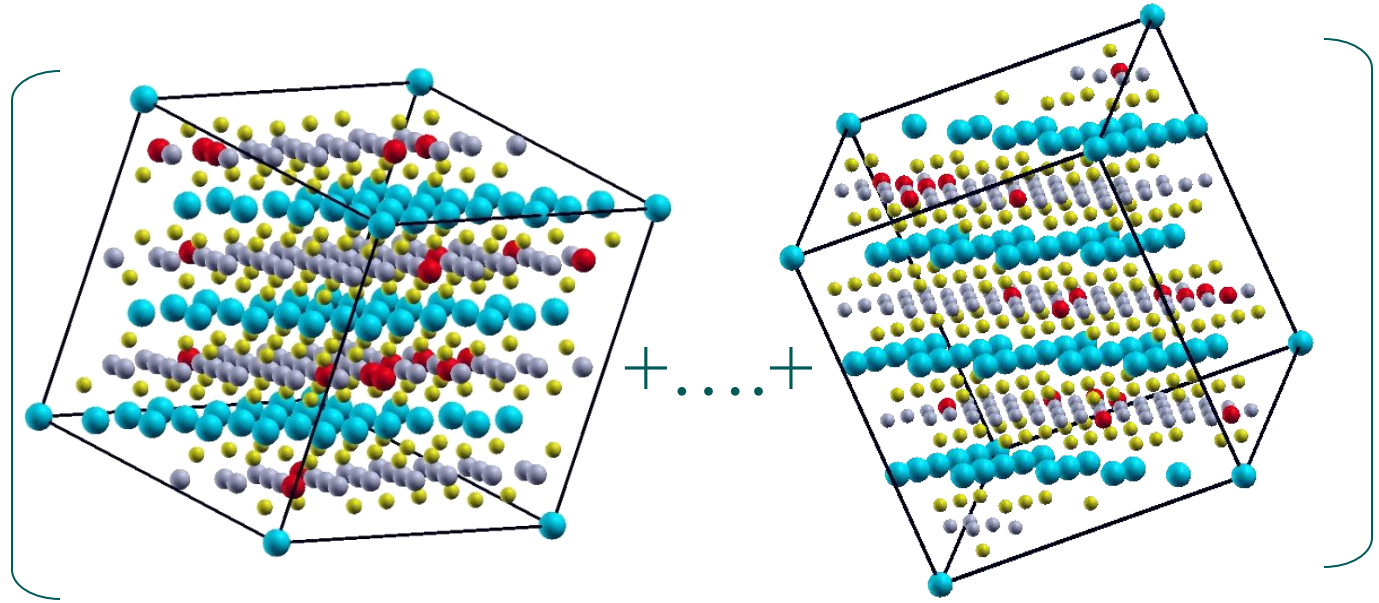


$$\langle G \rangle \approx P_1 G_1 + \dots + P_N G_N$$

Challenge: solving many Hamiltonians with big super cells

Configuration average over super cell

big super cells (100 ~ 1000 of atoms)



$$\langle G \rangle \approx P_1 G_1 + \dots + P_N G_N$$

Challenge: solving **many** Hamiltonians with **big** super cells

Constructing small and accurate effective H

Consider H from influence of impurities at x_1, \dots, x_N

$$H^{(x_1, \dots, x_N)} = H^0 + \sum_{i=1}^N \Delta^{(x_i)} + \sum_{i>j=1}^N \Delta^{(x_i, x_j)} + \dots$$

1. Calculate $H^0, H^{(x_i)}$ & $H^{(x_i, x_j)}$ in same Wannier basis from DFT
2. Calculate Δ by taking difference of H^{DFT} in Wannier basis

$$\Delta^{(x_i)} = H^{(x_i)} - H^0; \quad \Delta^{(x_i, x_j)} = H^{(x_i, x_j)} - H^0 - \Delta^{(x_i)} - \Delta^{(x_j)}$$

3. Assemble H_c for each disorder configuration

$$\begin{aligned} \left\langle r'n' \left| H^{((r_1, m_1), \dots, (r_N, m_N))} \right| r''n'' \right\rangle = \\ \left\langle r' - r'', n' \left| H^0 \right| 0n'' \right\rangle + \sum_{i=1}^N \left\langle r' - r_i, n' \left| \Delta^{(m_i)} \right| r'' - r_i, n'' \right\rangle + \dots \end{aligned}$$

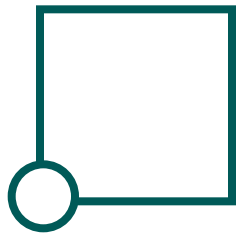
4. Solve H and unfold $A(k, \omega)$ to the normal cell unit
5. Average over configurations with different size & shape

$$\langle A(k, \omega) \rangle = \sum_c P_c A_c(k, \omega)$$

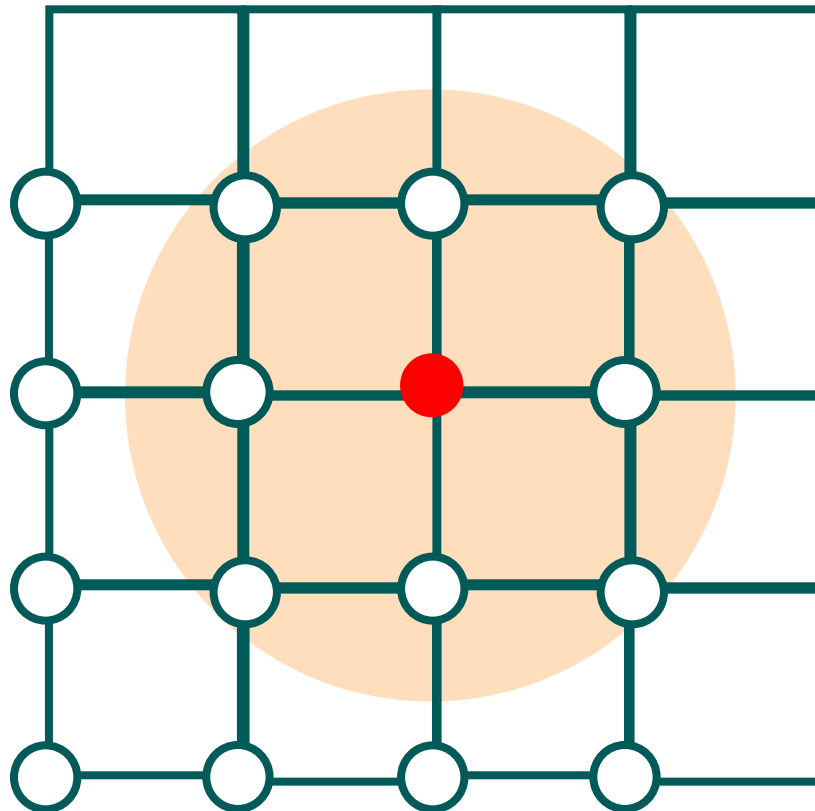
Including Effects in Density Functional Theory

two DFT Calculations

undoped normal cell



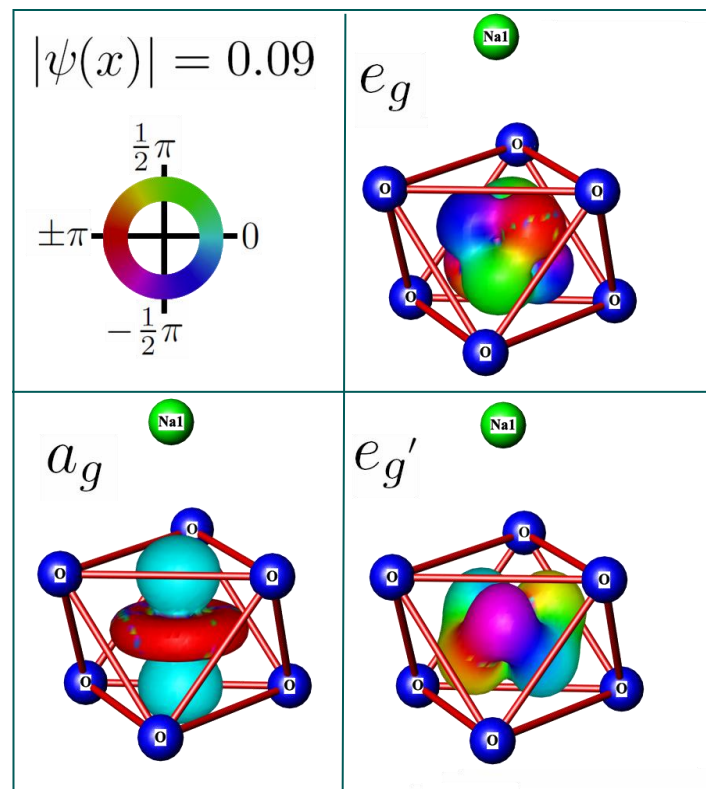
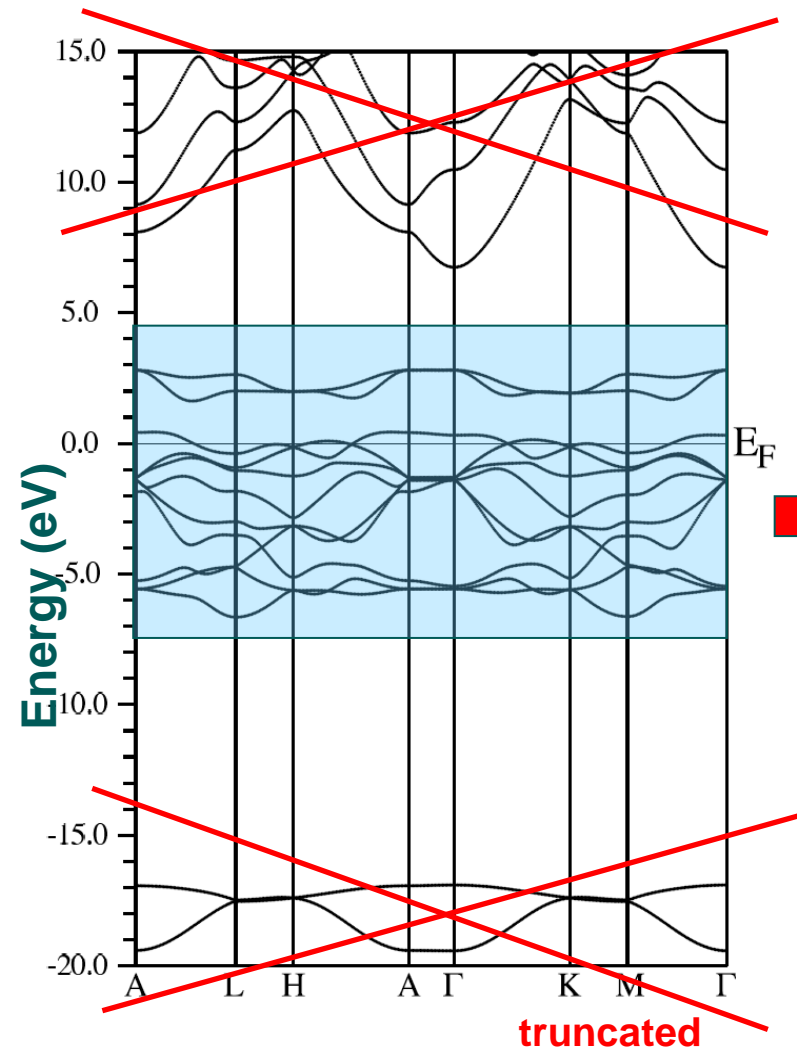
1 impurity in large super cell



Reducing Hilbert Space via Wannier Transformation

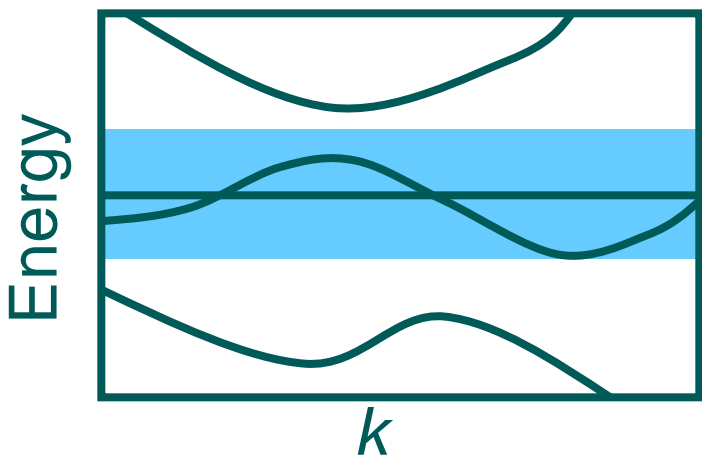
DFT CoO₂

Wannier functions

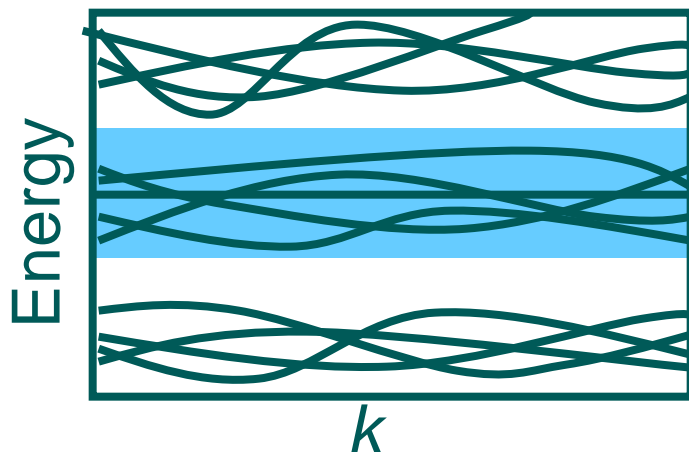


Obtain Effects of Impurities on H^{DFT}

DFT undoped



DFT 1 impurity



2 Wannier transformations

$$|\mathbf{Rn}\rangle = \sum_{\mathbf{k},j} e^{-i\mathbf{k}\cdot\mathbf{R}} \mathbf{U}_{nj}(\mathbf{k}) |\mathbf{k}j\rangle$$



Reduced Hilbert space

2 Tight Binding Hamiltonians

undoped

1 impurity

$$\mathbf{H}_{\text{DFT}}^0$$

$$\mathbf{H}_{\text{DFT}}^1$$

$$\Delta^{(x_i)} = H^{(x_i)} - H^0$$

Constructing small and accurate effective H

Consider H from influence of impurities at x_1, \dots, x_N

$$H^{(x_1, \dots, x_N)} = H^0 + \sum_{i=1}^N \Delta^{(x_i)} + \sum_{i>j=1}^N \Delta^{(x_i, x_j)} + \dots$$

1. Calculate $H^0, H^{(x_i)}$ & $H^{(x_i, x_j)}$ in same Wannier basis from DFT
2. Calculate Δ by taking difference of H^{DFT} in Wannier basis

$$\Delta^{(x_i)} = H^{(x_i)} - H^0; \quad \Delta^{(x_i, x_j)} = H^{(x_i, x_j)} - H^0 - \Delta^{(x_i)} - \Delta^{(x_j)}$$

3. Assemble H_c for each disorder configuration

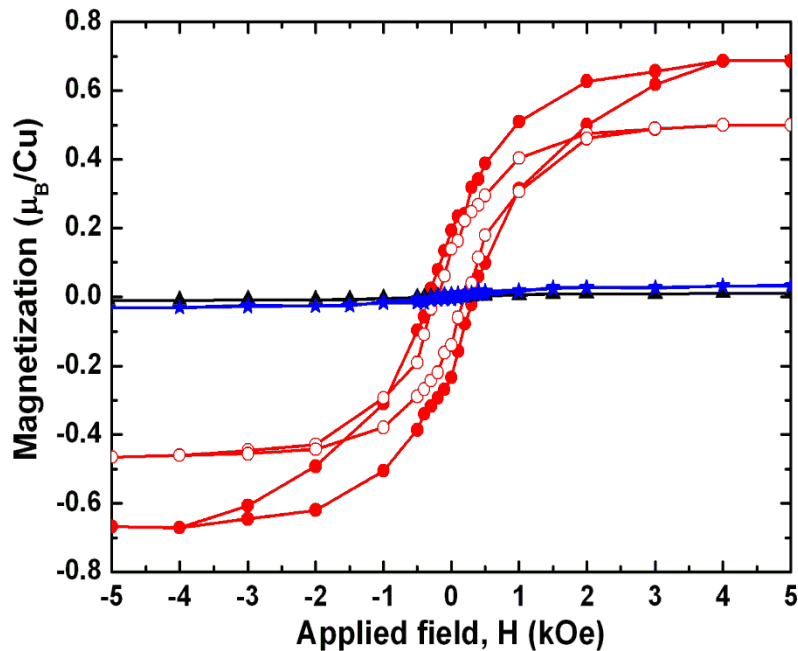
$$\begin{aligned} \left\langle r'n' \left| H^{((r_1, m_1), \dots, (r_N, m_N))} \right| r''n'' \right\rangle = \\ \left\langle r' - r'', n' \left| H^0 \right| 0n'' \right\rangle + \sum_{i=1}^N \left\langle r' - r_i, n' \left| \Delta^{(m_i)} \right| r'' - r_i, n'' \right\rangle + \dots \end{aligned}$$

4. Solve H and unfold $A(k, \omega)$ to the normal cell unit
5. Average over configurations with different size & shape

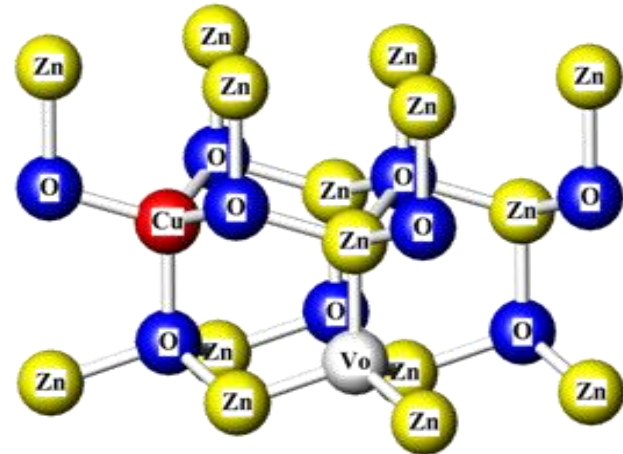
$$\langle A(k, \omega) \rangle = \sum_c P_c A_c(k, \omega)$$

Substitution & vacancies: $\text{Zn}_{1-x}\text{Cu}_x\text{O}_{1-y}$

SQUID



- ▲ ZnO @ 300K (O-poor)
- ★ ZnO:Cu @ 300K (O-rich)
- ZnO:Cu @ 5K (O-poor)
- ZnO:Cu @ 300K (O-poor)

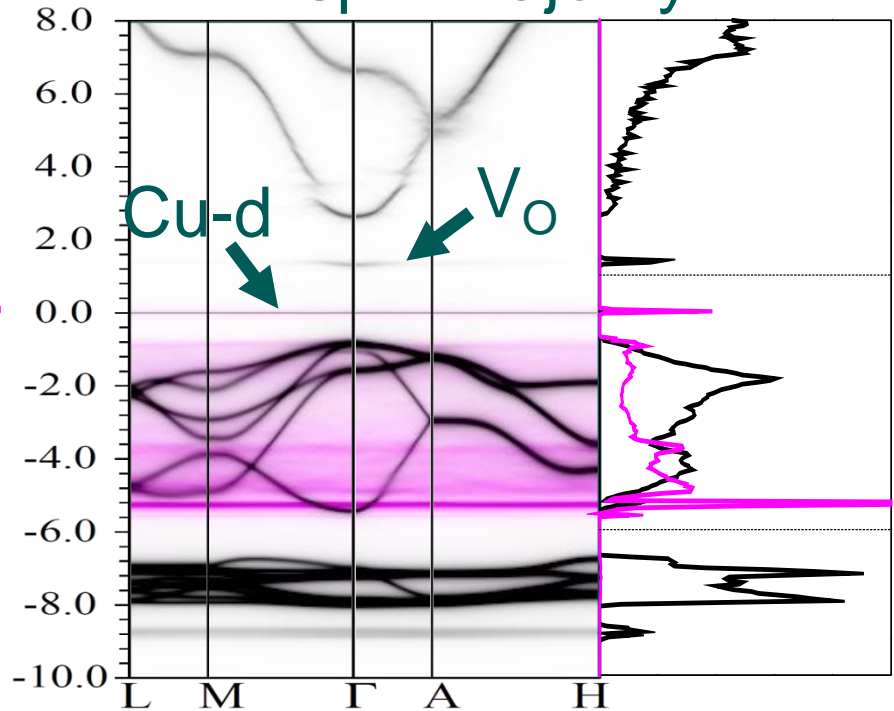
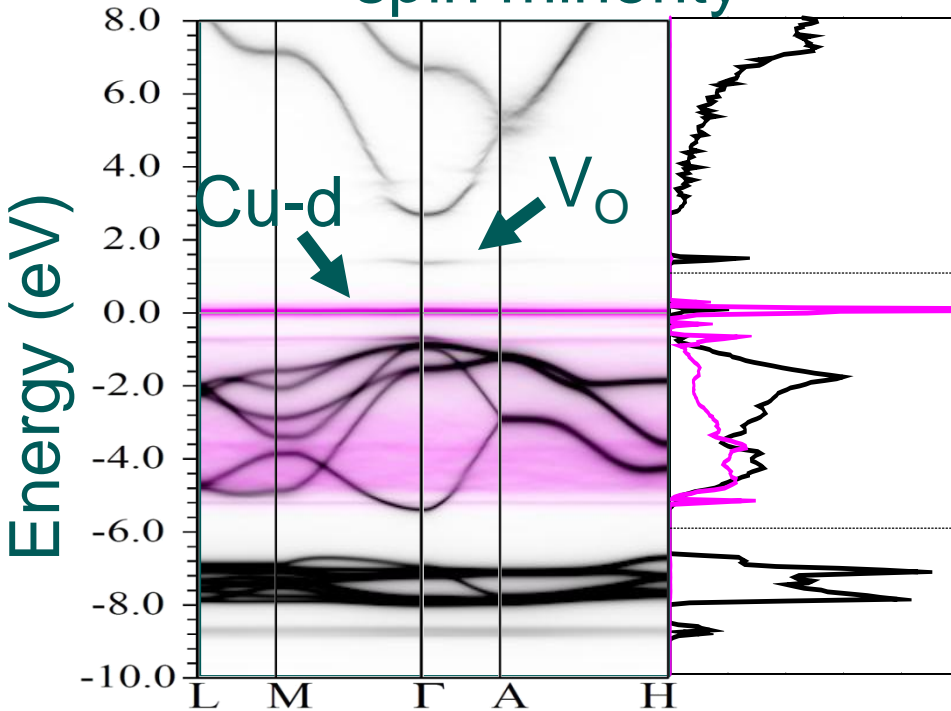


Q1) Where do the doped electrons reside?

Q2) What is the role of oxygen vacancy?

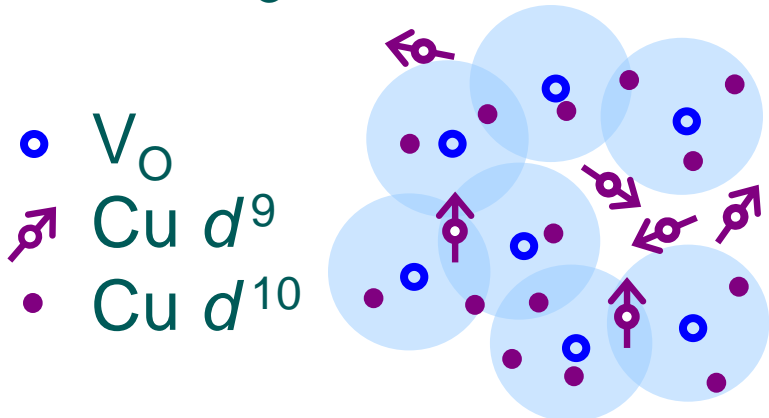
spin minority

spin majority



A1) Doped electrons reside in Cu orbitals

A2) V_O mediates Cu moments





Do transition metal substitution dope carriers in Fe-based superconductors

T. Berlijn & Wei Ku

PRL **108**, 207003 (2012)

Do TM substitutions dope carriers in Fe-SC?

Periodic Table of the Elements © www.elementsdatabase.com

1 H																	2 He														
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne														
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar														
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr														
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe														
55 Cs	56 Ba	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89 Ac	104 Unq	105 Unp	106 Unh	107 Uns	108 Uno	109 Une	110 Uun																						
																		119	120											121	122
																		119 Og	120 Nh											121 Ts	122 Lr

118
Og

119
Ts

120
Nh

121
Ts

122
Lr

26 Fe	27 Co	28 Ni	29 Cu	30 Zn
44 Ru	45 Rh	46 Pd	47 Ag	48 Cd

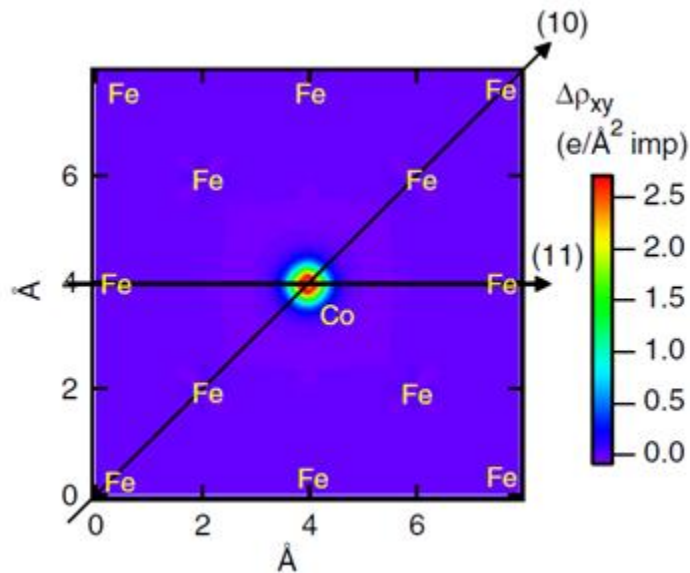
more valence electrons than Fe → expect electron doping

Additional Charge Residing in the Substituted Atoms in Fe-SC

Theory

DFT (density)

Co:122

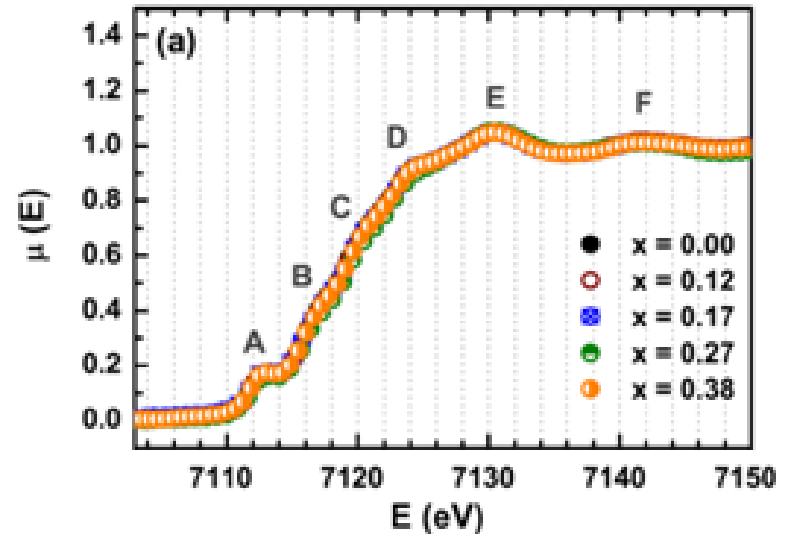


H. Wadati *et al*, PRL **105** (2010)

Experiment

XANES

Co:122

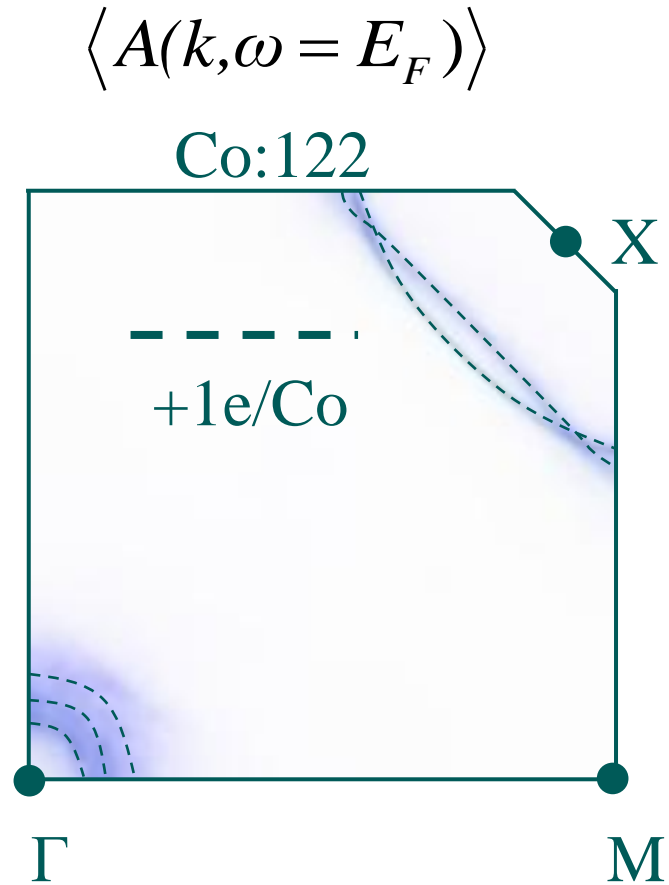


E. M. Bittar *et al*, PRL **107** (2011)

- Substitution does not dope the system (?)

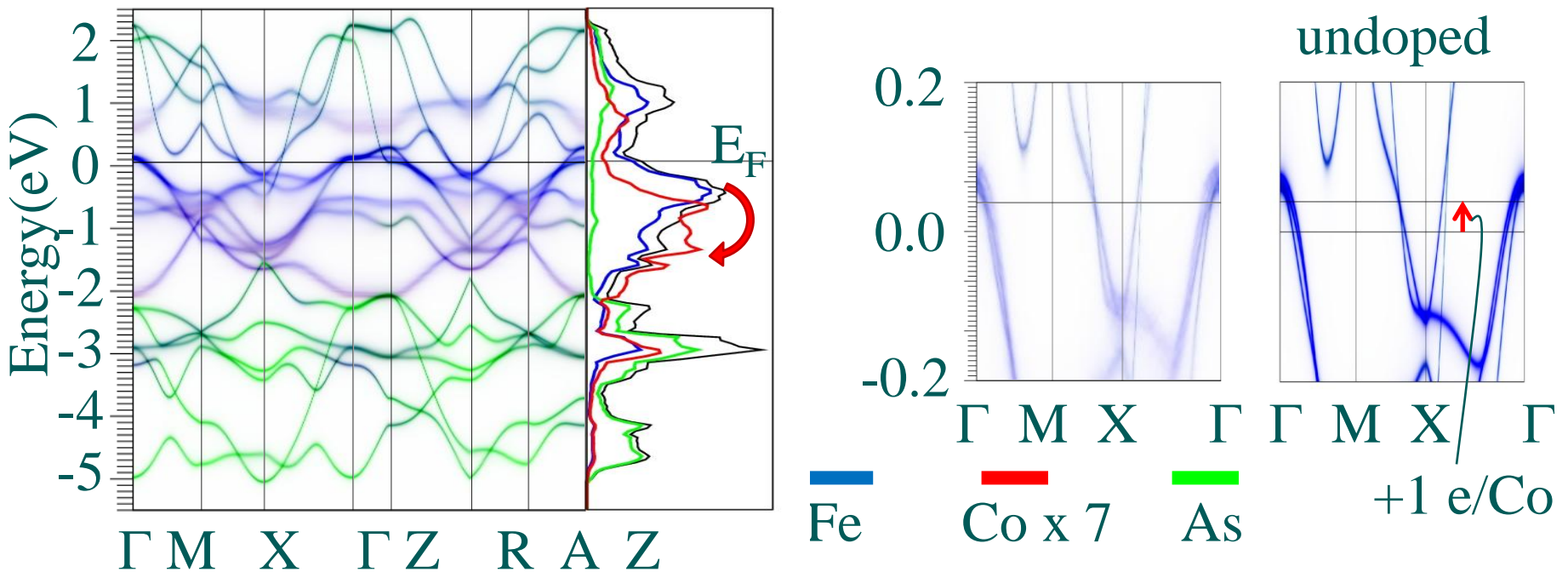
12.5% Co Substituted BaFe₂As₂

	α	$\sum_{n \in \alpha} \int_{BZ} d^3k \int^{E_F} d\omega \langle A_n(k, \omega) \rangle$
122	Fe	7.22
Co:122	Fe	7.19 x 7/8
Co:122	Co	8.40 x 1/8

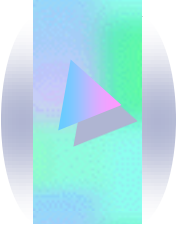


- Change of density in substituted atoms \rightarrow not doping ?
- Enlarged Fermi surfaces & shift of chemical potential \rightarrow doping?

Example: Does Co/Zn substitution dope Fe-SC?



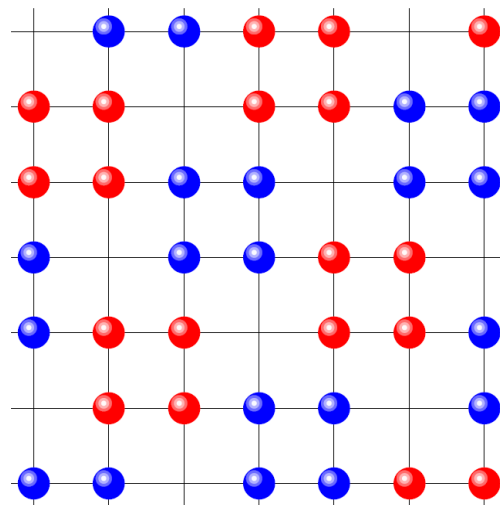
Most definitely yes, in the simplest manner,
but some carriers lose coherence!



Fe vacancy in $K_2Fe_4Se_5$

T. Berlijn, P. Hirschfeld, & Wei Ku

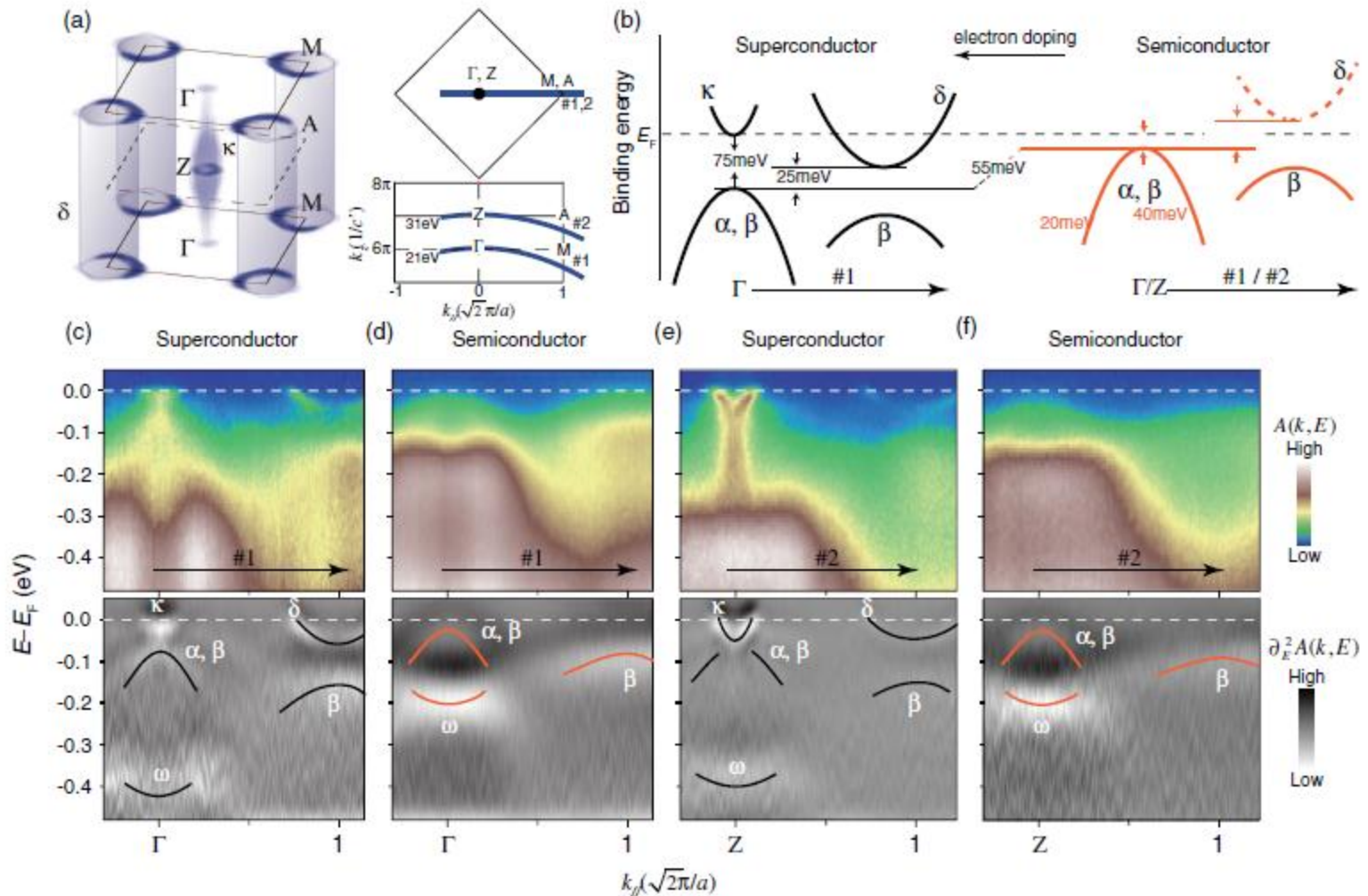
PRL **109**, 147003 (2012)



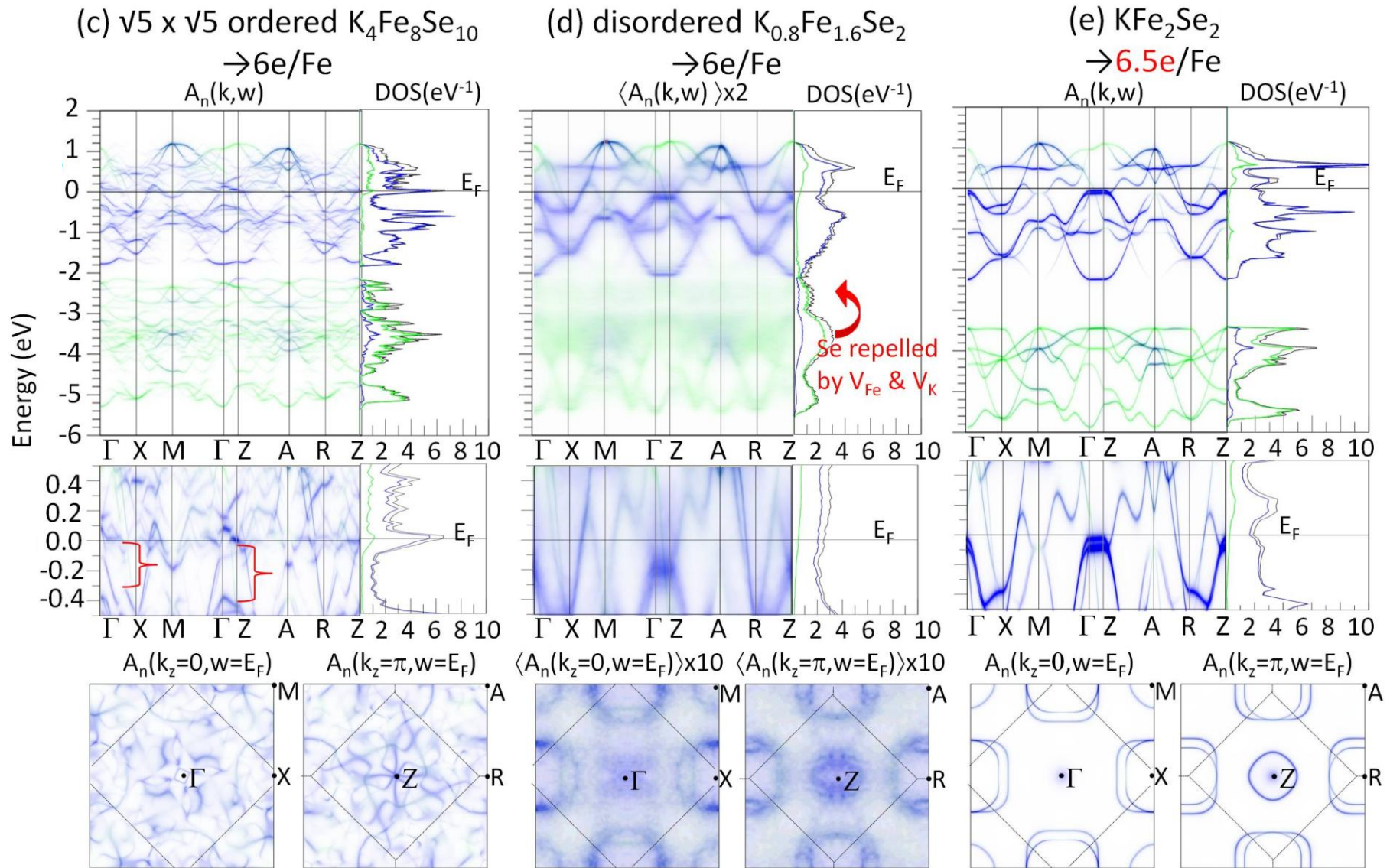
A heavily electron doped system?

ELECTRONIC IDENTIFICATION OF THE PARENTAL ...

PHYS. REV. X 1, 021020 (2011)



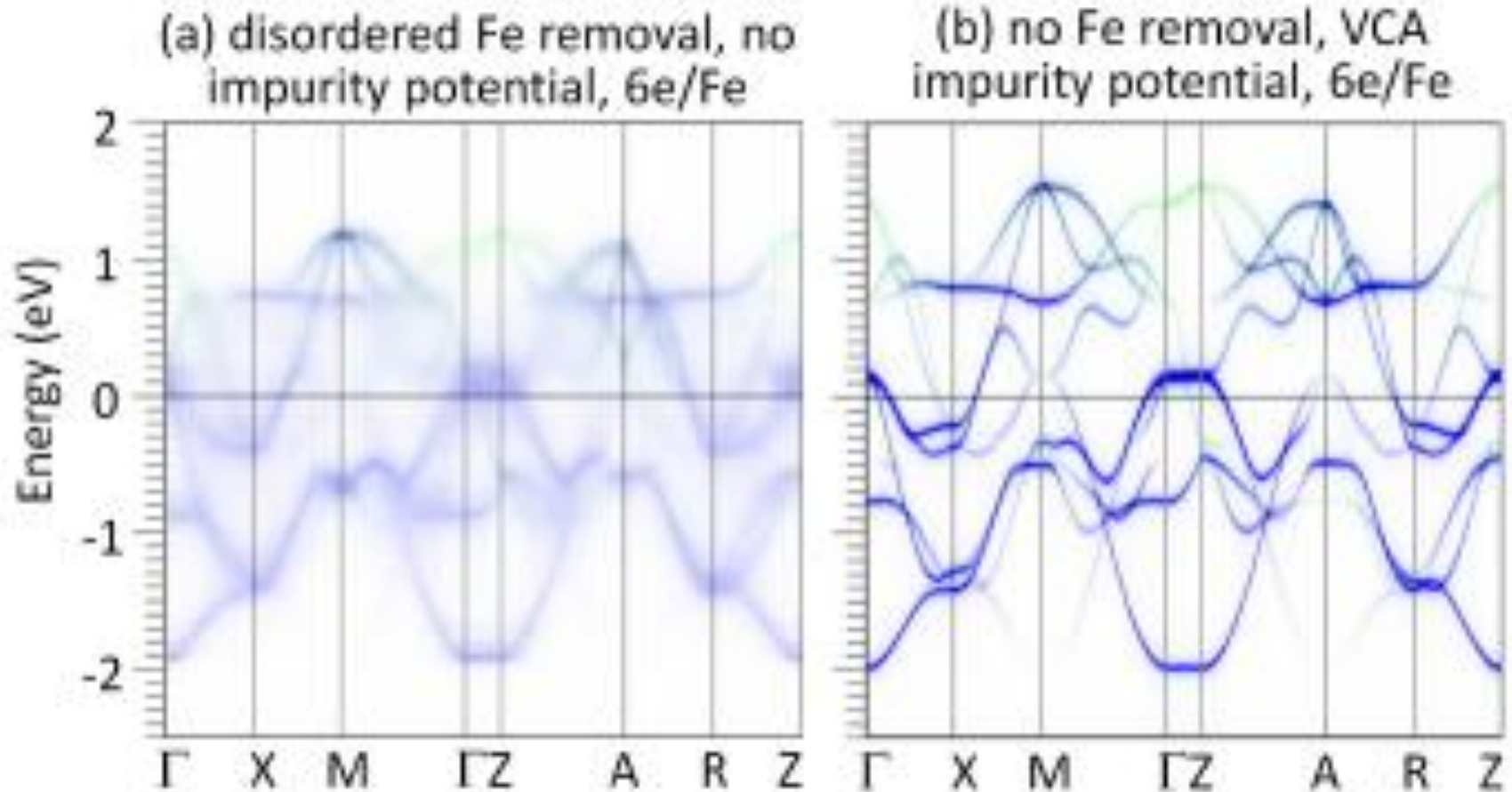
Effective “doping” with Fe vacancy: Luttinger theorem?



Appears to be heavily doped $\sim 0.5 e / Fe$ with disordered Fe vacancy

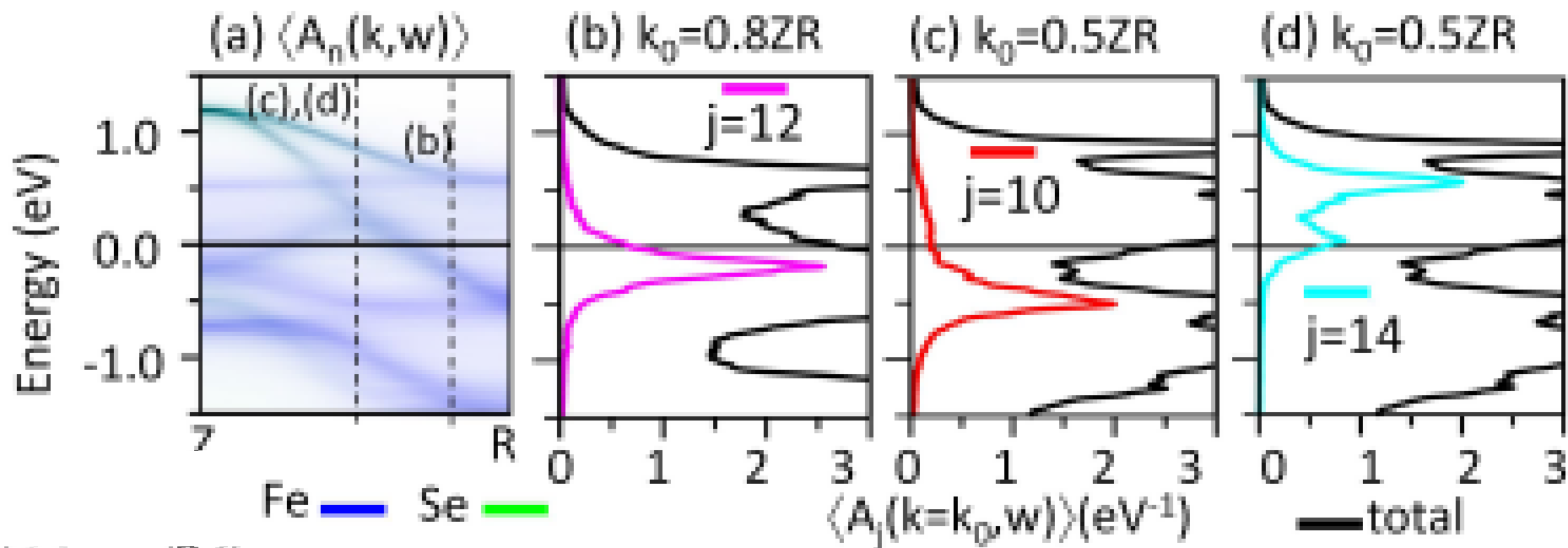
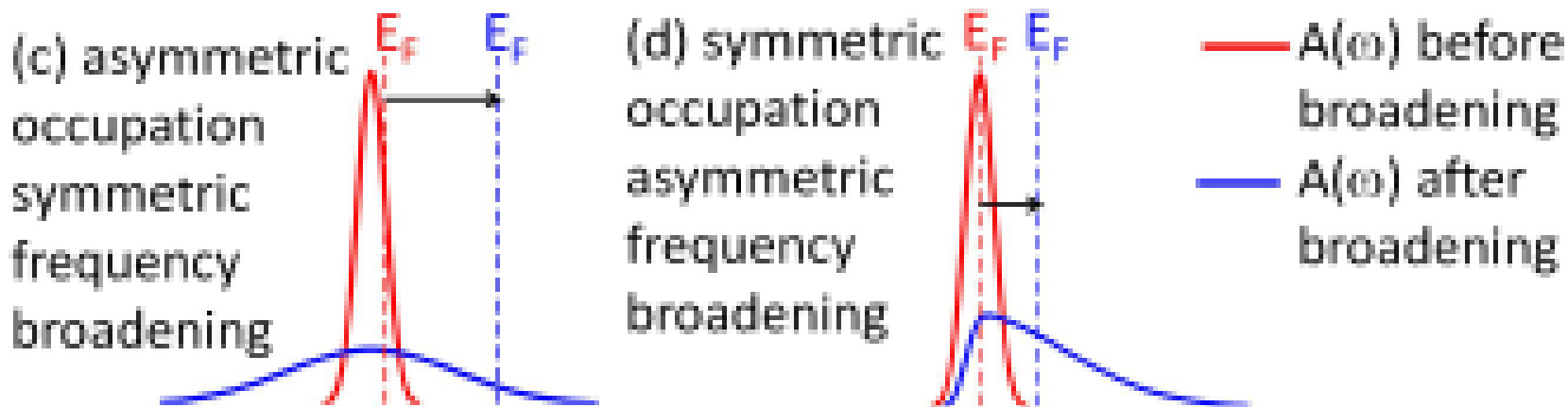
Tom Berlijn, Peter Hirschfeld, and Wei Ku, **109**, 147003 (2012)

Origin of the effective “doping” with Fe vacancy



- Need both disorder and the impurity potential

Effects of disordered impurity potential





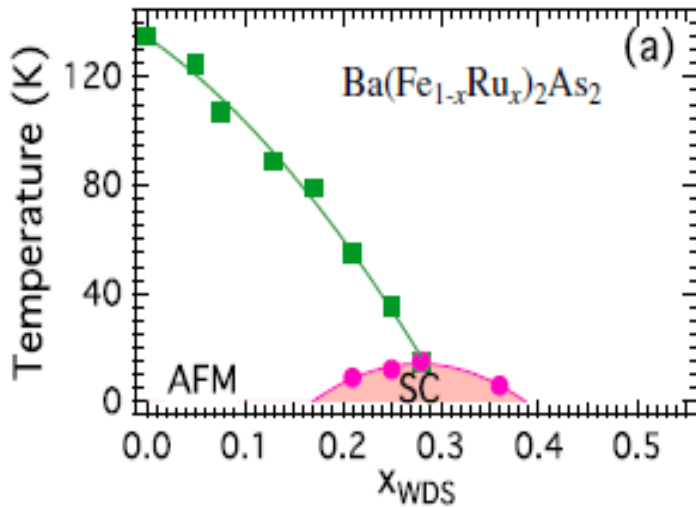
Ru substitution of BaFe₂As₂

Limin Wang, Tom Berlijn, Chia-Hui Lin, Yan Wang, Peter Hirschfeld,

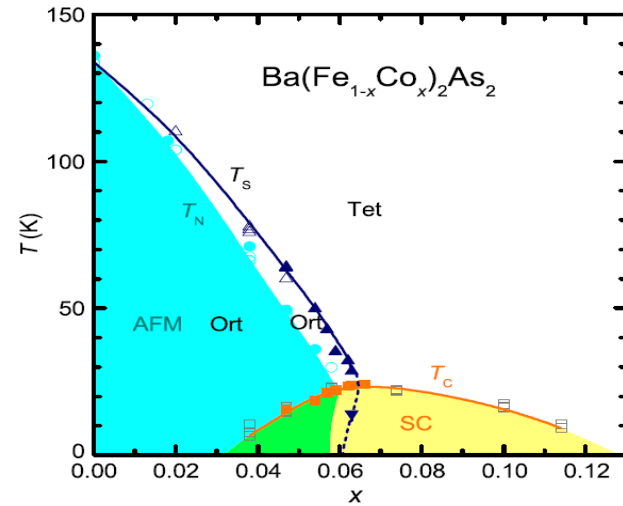
Wei Ku

PRL 110, 037001 (2013)

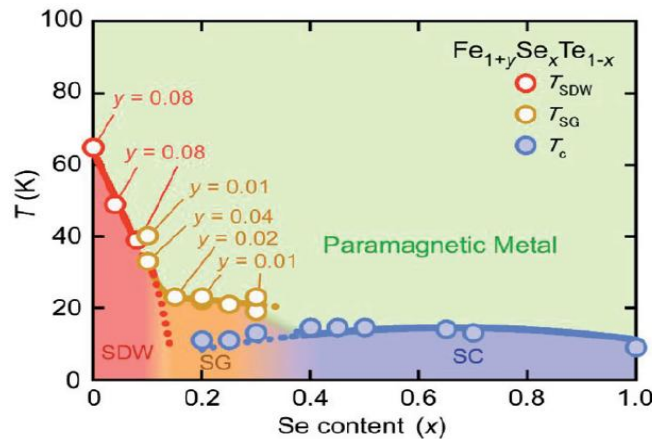
Temperature-composition phase diagram



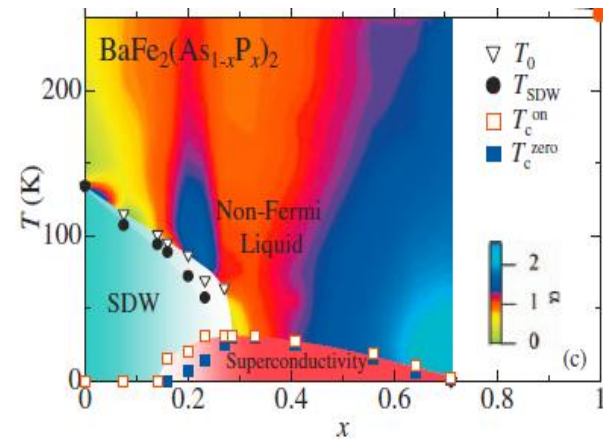
[R. S. Dhaka *et al*, PRL 107, 267002]



S. Nandi *et al*, PRL 104, 057006



[N. Katayama, *et al*, JPSJ 79, 113702]

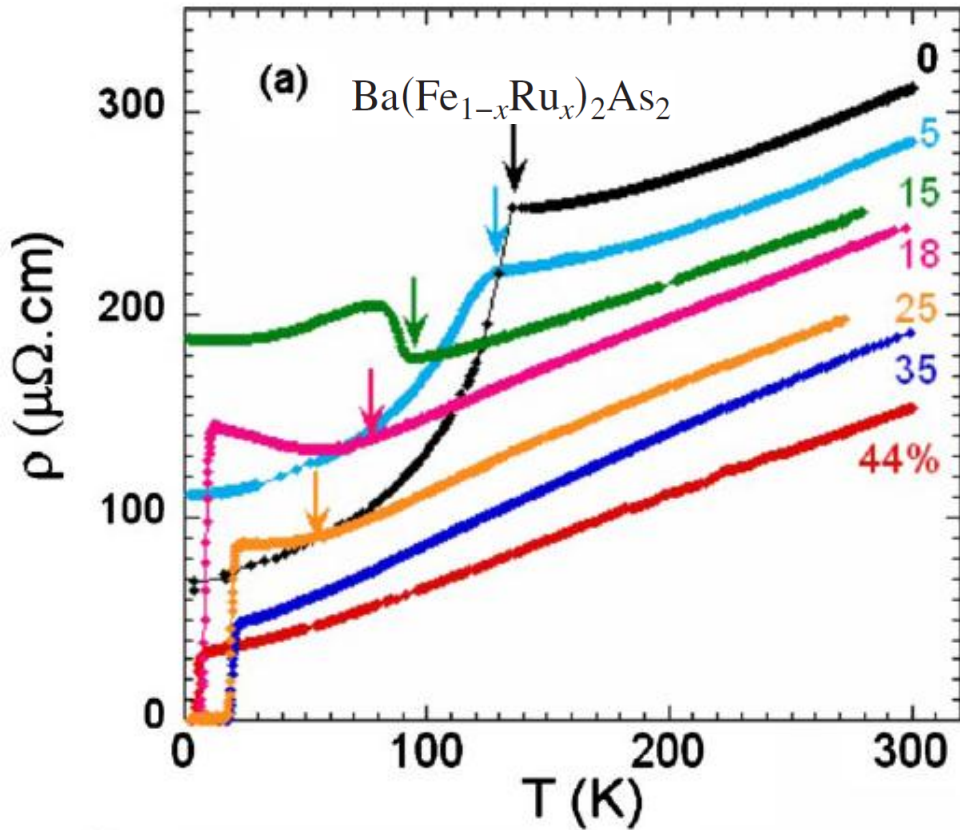


[S. Kasahara, *et al*, PRB 81 184519]

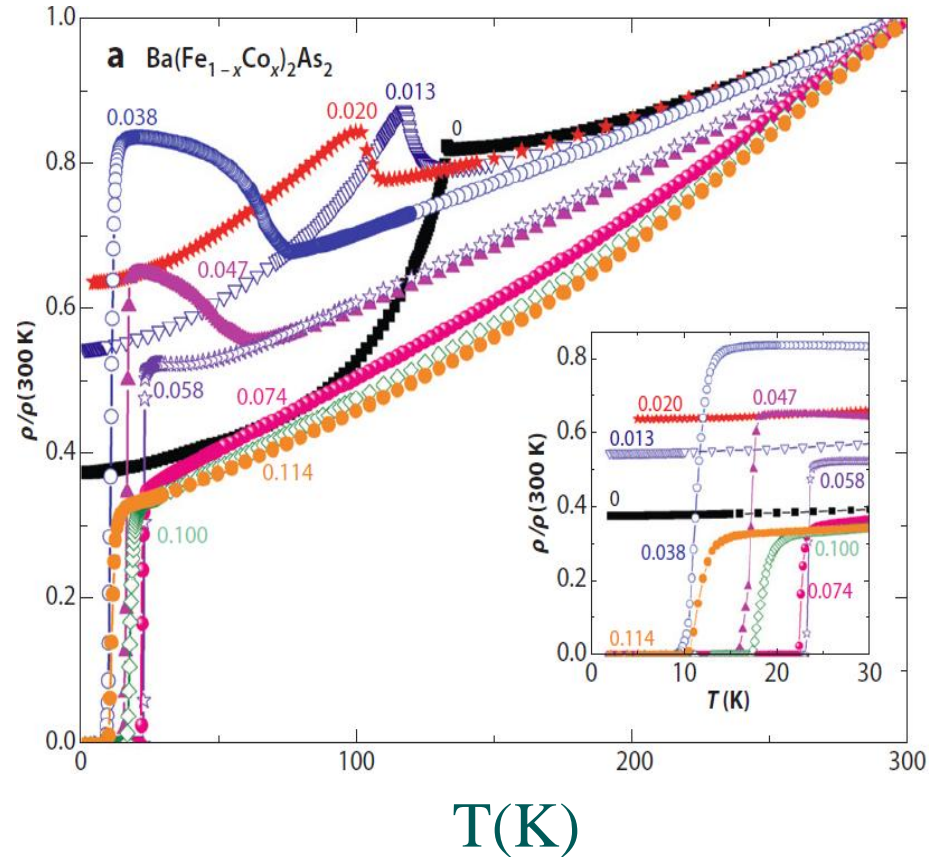
Ru/Co atoms replace the most essential Fe atom.
Superconductivity can survive even with 40% Ru substitution

Transport property

[F. Rullier-Albenque et al., PRB 81, 224503 (2010)]



[P. C. Canfield et al., Annu. Rev. Cond. Matt. 27 (2010)]

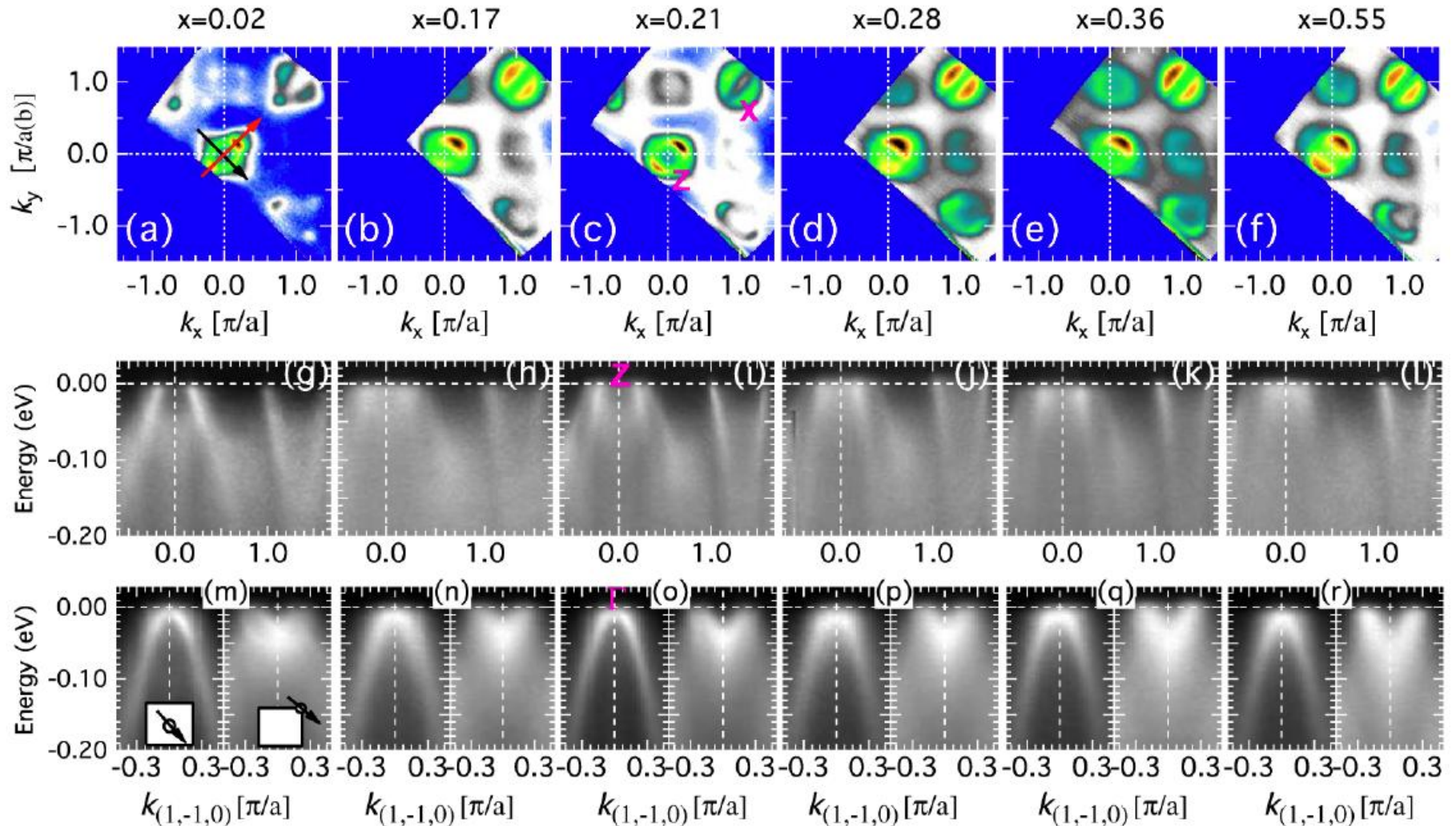


Ru substituted samples exhibit a residual resistivity comparable to 8% substituted system at a much higher 35% substitution level

Experimental (ARPES) controversy

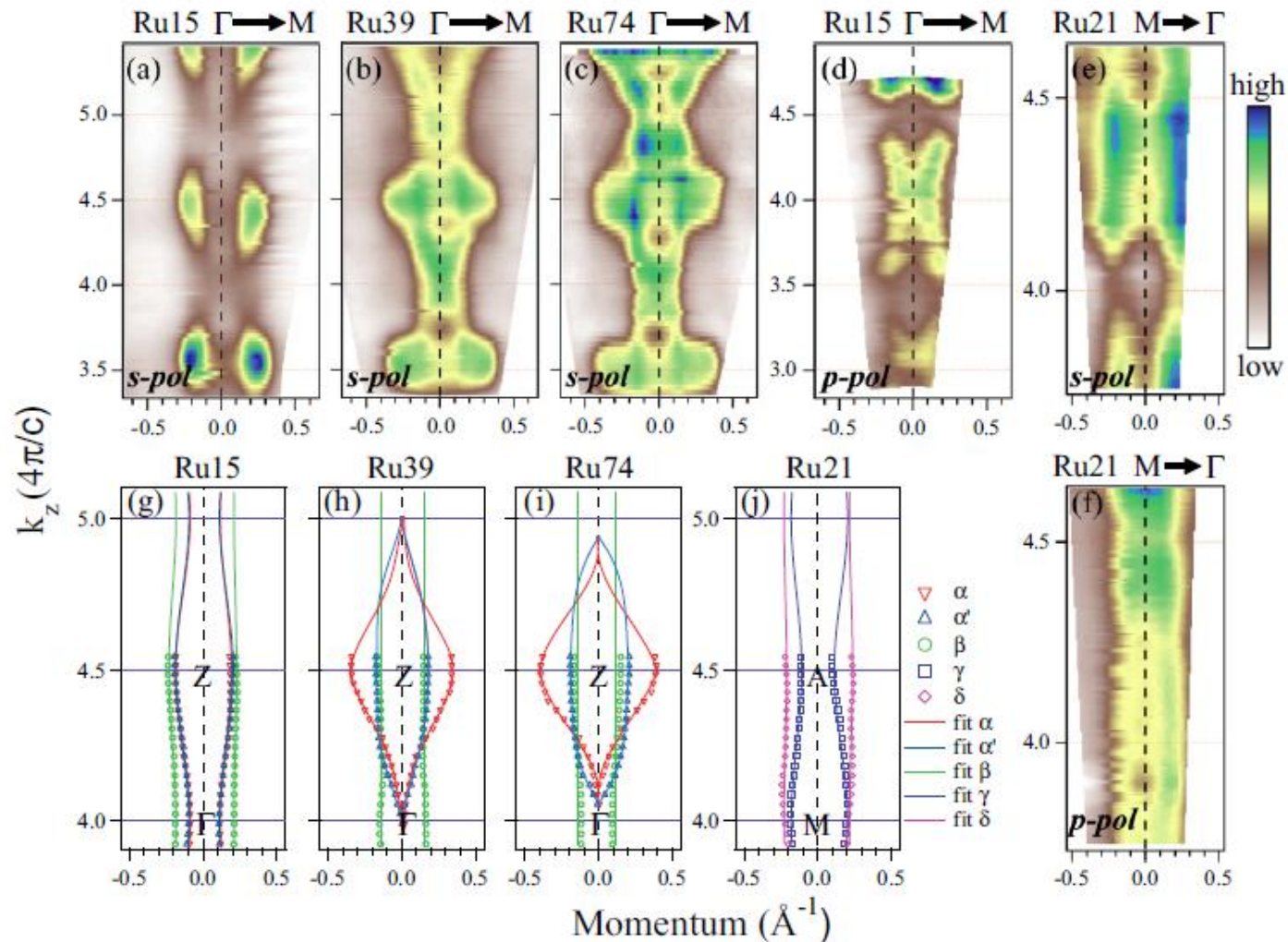
- Nearly substitution-independent FS

[R. S. Dhaka *et al*, PRL 107, 267002(2011)]

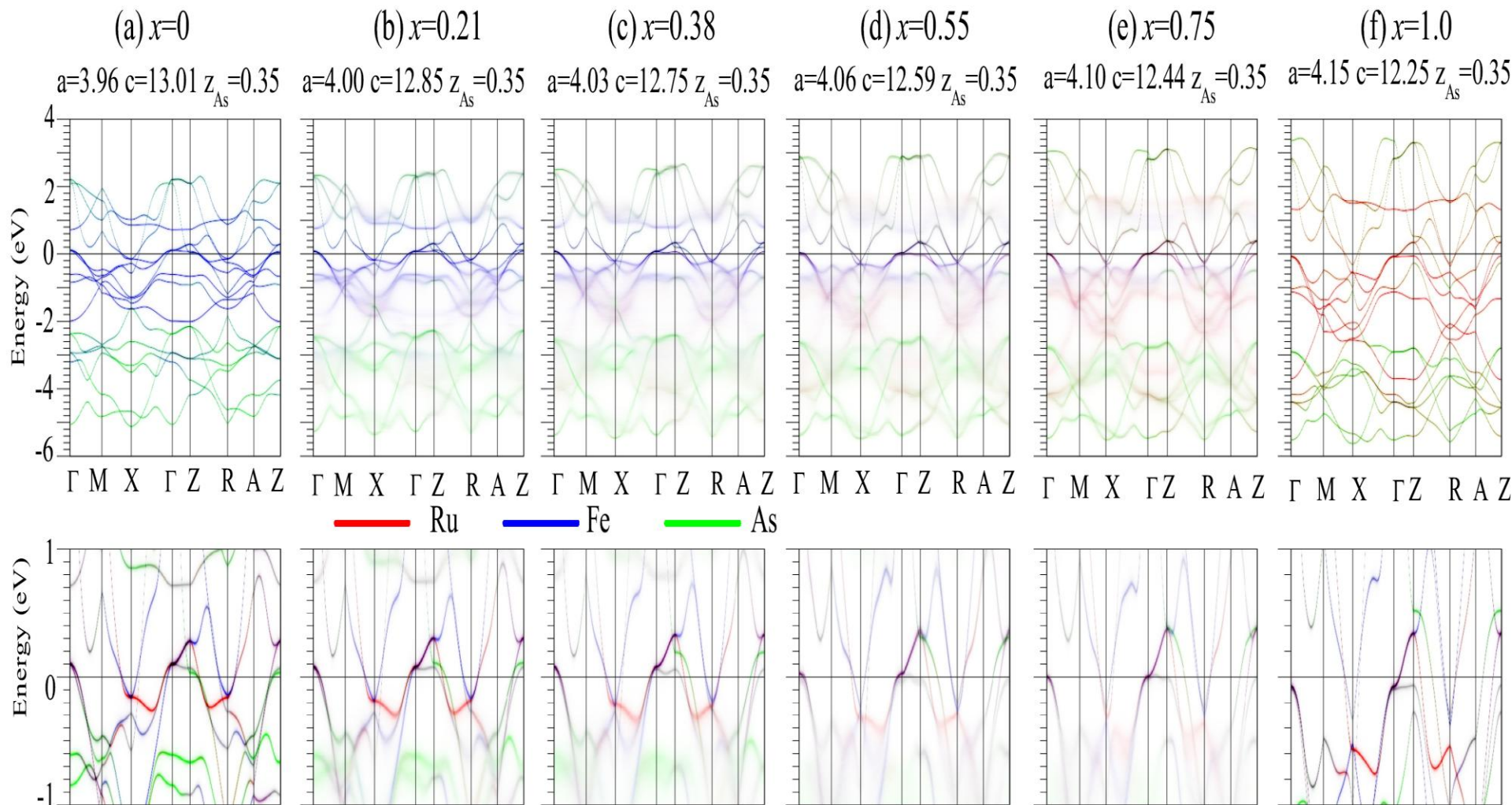


Experimental (ARPES) controversy

- A crossover from two-dimensional to three dimensional structure [N. Xu et al, PRB 86, 064505]

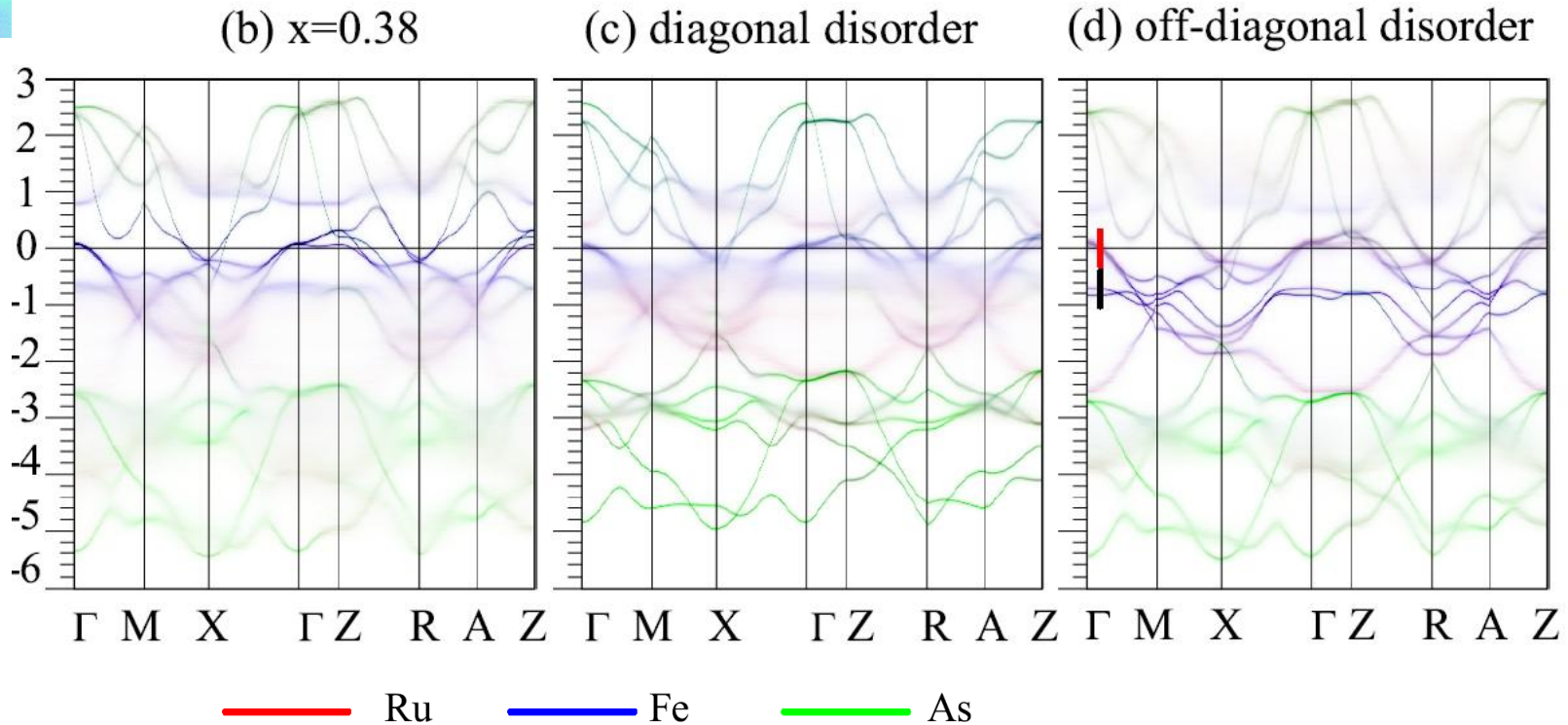


Band structure for different Ru substitution



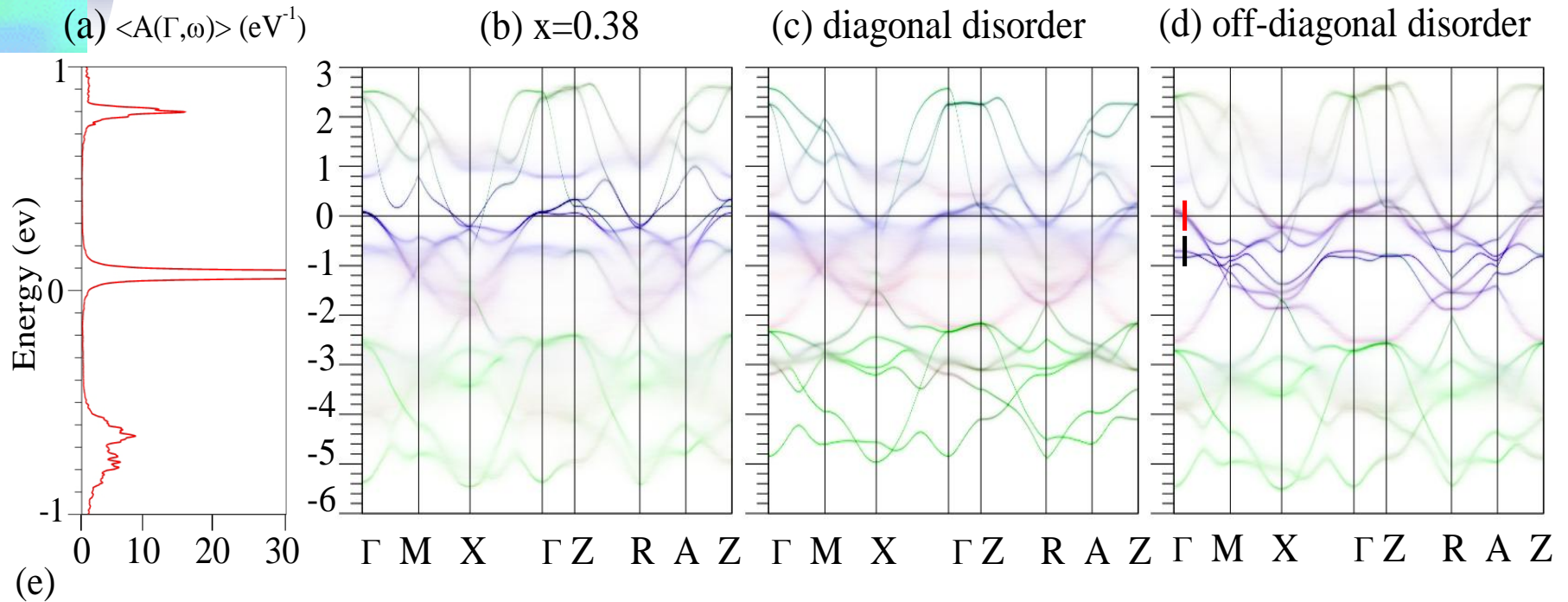
- The bands are heavily smeared. (-0.8eV)
- $x=0.55$ is the most disordered one.
- Fe bands around the Fermi level remain very sharp.

Origin of the superdiffusion around the Fermi level



- diagonal disorder only : strong smearing of the entire d -band
- off-diagonal disorder only : scattering diminishes near the center of the d -band.
- combined \rightarrow weak impurity scattering near E_F

Origin of the superdiffusion around the Fermi level



k	j	k'	j'	$\langle kj \Delta k'j' \rangle$	$\langle kj \Delta_D k'j' \rangle$	$\langle kj \Delta_{OD} k'j' \rangle$
Γ	9	$0.3\Gamma M$	10	-0.0578	-0.3023	0.2445
Γ	8	$0.3\Gamma M$	7	-0.2938	-0.3316	0.0378

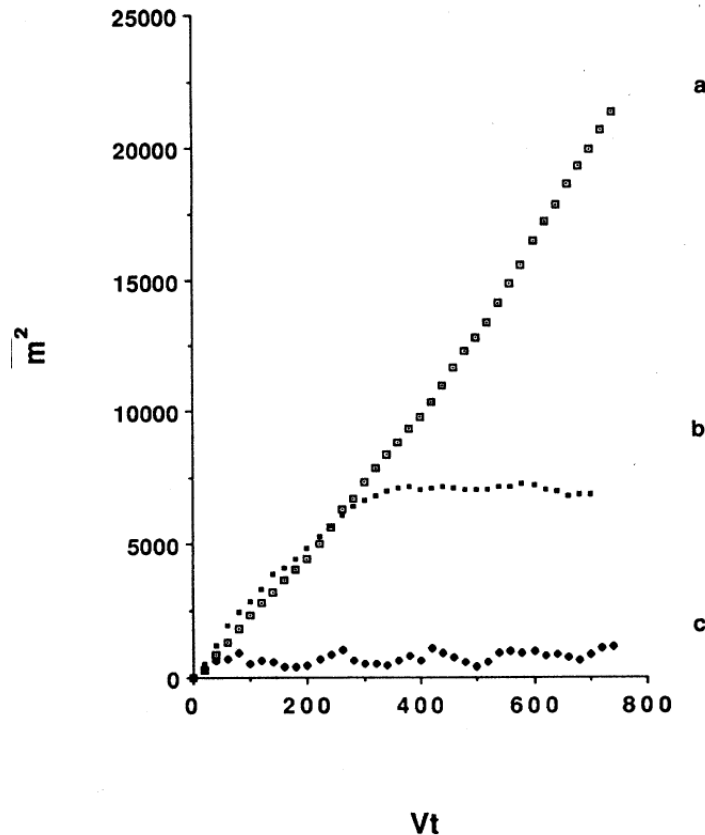
- The insensitivity originates from a coherent interference between on-site and off-site impurity effects.

Superdiffusion in 1D model

[D. H. Dunlap, K. Kundu, and P. Phillips, PRB 40, 10999 (1989)]

Single-particle Hamiltonian

$$\begin{aligned}
 H = H_{\text{lattice}} + \sum_{\mathbf{k}} \epsilon_0 a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + \sum_{\mathbf{k}, \mu} 2V_{\mu} \cos(\phi_{\mu} - k_{\mu}) a_{\mathbf{k}}^\dagger a_{\mathbf{k}} \\
 + \sum_{\mathbf{k}, \mathbf{q}, \mu} 2i [\gamma_{\mu} \sin(k_{\mu} - \theta_{\mu}) - \gamma_{\mu} \sin(k_{\mu} - q_{\mu} - \theta_{\mu}) \\
 - G_{\mu} \sin(q_{\mu})] x_q^{\mu} a_{\mathbf{k}}^\dagger a_{\mathbf{k} - \hat{\mu}q_{\mu}}, \quad (1.2)
 \end{aligned}$$



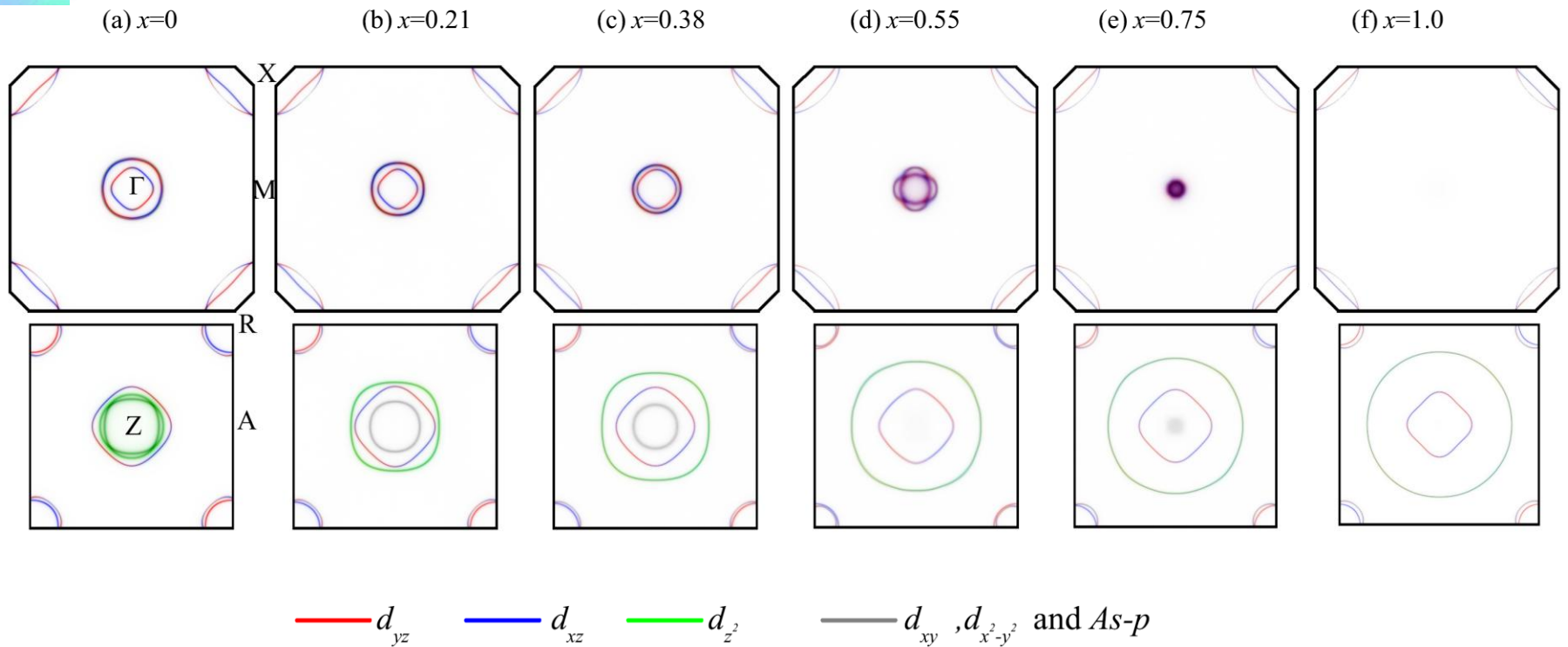
a correlated disorder ($\gamma = G$)

b site-diagonal disorder.

c site-off diagonal disorder.

- Correlations between site and off-diagonal disorder give rise to superdiffusion transport.

Fermi surface (2Fe) for different Ru substitution



- The hole pockets become more three dimensional
- The electron pockets show a slight substitution dependence.

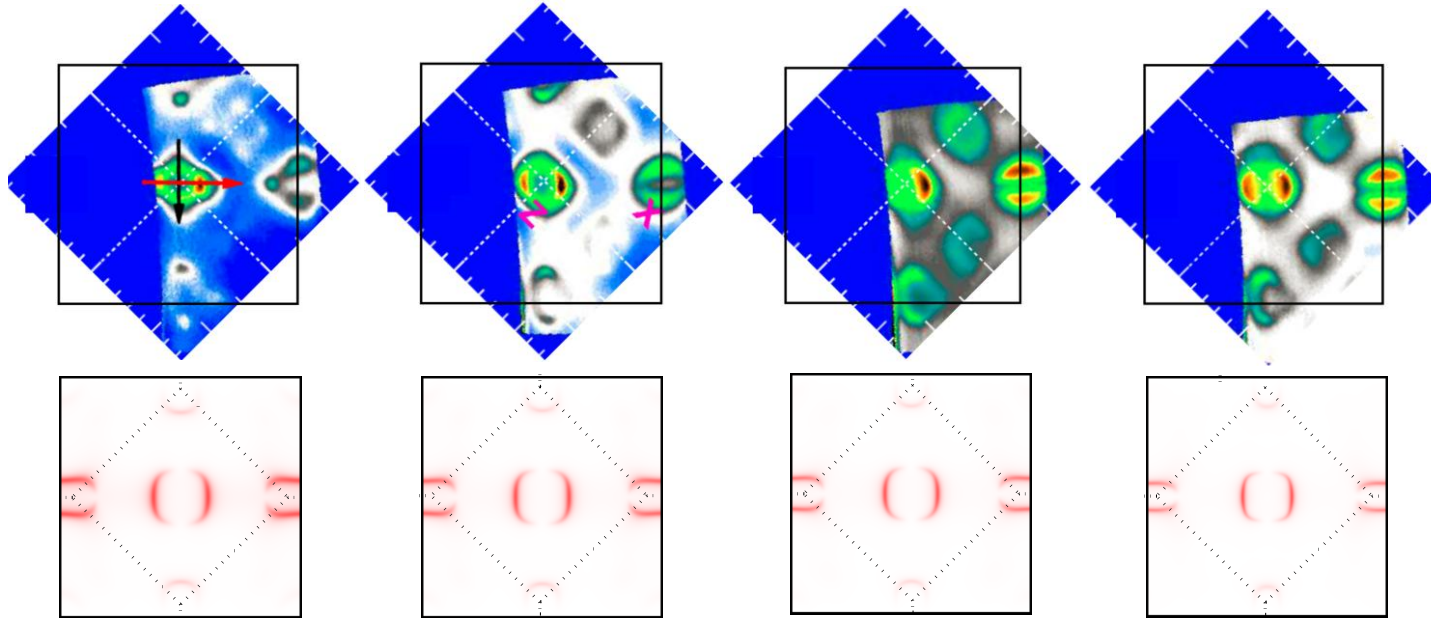
Resolution of the ARPES controversy

(a) $x=0.02$

(b) $x=0.21$

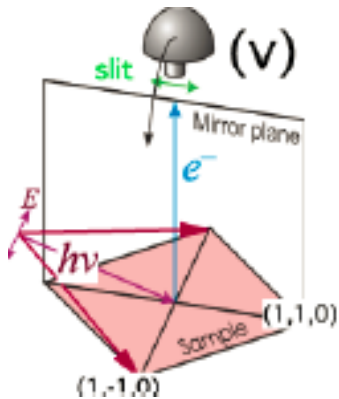
(c) $x=0.36$

(d) $x=0.55$



[R. S. Dhaka *et al*,
PRL 107, 267002]

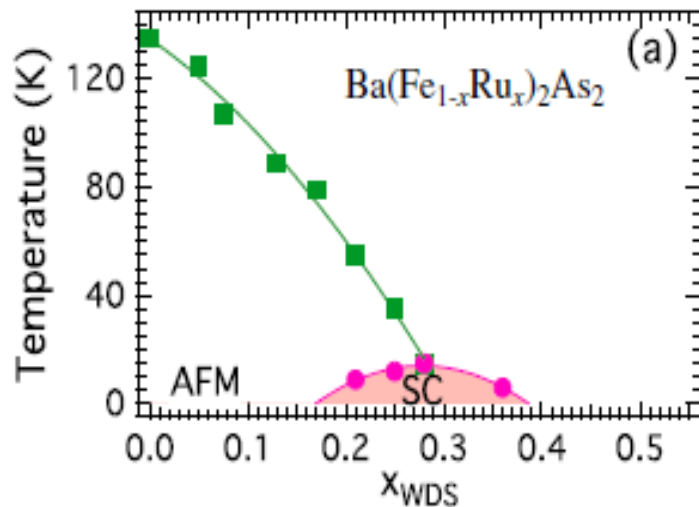
1Fe zone



- This photon polarization couples mostly to the yz orbital.
- The yz character of our unfolded BS also show little change.

Suppression of the long-range order

	$x = 0$	$x = 0.21$	$x = 0.38$	$x = 0.55$	$x = 0.75$	$x = 1.0$
$e/\text{Fe} = h/\text{Fe}$	0.162	0.151	0.142	0.127	0.123	0.120
$\text{DOS}(E_F)$	2.393	2.024	1.775	1.458	1.299	0.868



- Larger splitting between the conduction and valence bands \rightarrow carrier reduction
- The reduced DOS & enhanced 3D nature \rightarrow suppress the long-range orbital/magnetic order

[R. S. Dhaka *et al*, PRL 107, 267002]



Conclusions

- Symmetry-respecting Wannier function
 - Simple, flexible and efficient basis to describe physics
 - Intuitive microscopic picture + quantitative realism
- Unfolding 1st-principles band structures
 - Visualizing each band's coupling to the order parameters and other symmetry breakers
 - Useful for the study of extrinsic (doping, intercalation, vacancy, substitution, & defects) and intrinsic (CDW, SDW, OO, & structure distortion) effects on the electronic structure
 - Direct comparison with ARPES spectrum, built-in matrix elements effects
- A non-perturbative beyond-mean-field first-principles method for materials with disordered impurities
 - Defects, vacancy, chemical substitution, doping & intercalation
 - Thermal/quantum fluctuation induced spatial inhomogeneity
 - Paramagnetic phase with disordered local moments