TWO-SPECIES BOSE HUBBARD MODEL

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Cold Atoms

On Surface of Sun: Miss many aspects of nature

Surface of Earth: Different states of matter: solid, liquid and gas

Further Cooling: In Kelvin: Superconductivity (1911)
Superfluidity in Liquid Helium (4He) (1938)
In milli-Kelvin: Superfluidity in 3He (1972)

Laser cooling/evaporative cooling(1980):
In nano-Kelvin: Bose-Einstein Condensation (1995)
(This was predicted for photons in 1924 by Bose and Einstein)
These achievements recognized with Nobel prizes!
Optical Lattices and Bose Hubbard Model

- **Optical lattices:** Artificial crystal of lights consisting of thousands of optical micro traps: created by interfering of laser beams!

- Bosons loaded in optical lattices undergo a superfluid-Mott insulator transition: Theoretically proposed and verified experimentally!

- The advance of optical lattice experiments and the experimental tunability of Hamiltonian parameters using laser and magnetic fields allow the realization of strongly correlated models.

D. Jaksch et al., PRL(1998)
M. P. A. Fisher et al., PRB(1989)
M. Greiner et al., NATURE(2002)
The superfluid phase is characterized by Bose-Einstein condensation, gapless excitations and finite compressibility in the weak coupling regime.

The Mott insulator phase is characterized by commensurate occupations, gapped excitations and incompressibility in the strong coupling regime.

Mixtures of atoms can give rise to even more interesting and complex phases.

\[ H = -t \sum_{\langle ij \rangle} (a_i^+ a_j + \hbar c) + \frac{U}{2} \sum_i n_i (n_i - 1) ; \quad n_i = a_i^+ a_i \]
Two species hard core bosonic model

\[ H = -t_a \sum_{\langle ij \rangle} (a_i^+ a_j + hc) - t_b \sum_{\langle ij \rangle} (b_i^+ b_j + hc) + U_{ab} \sum_i n_{i,a} n_{i,b} \]

- Commutation rules on different sites!
- Anti-commutation Rules on identical site

Our focus is to study the model as a function of doping.

1. Equal populations \((N_a = N_b)\) and mass imbalance \((t_a < t_b)\).
2. Population imbalance \((N_a \neq N_b)\) and mass imbalance \((t_a < t_b)\).

Method: QMC simulations using the Stochastic Green function algorithm

V.G. Rousseau, PRE. 78, 056707 (2008), and arXiv:1209.0946
Doping results with mass imbalance between species

\[ t_a = 0.08t \]
\[ t_b = 1.0t \]
\[ U_{ab} = 6t \]

\[ \rho = \frac{N}{L^2} \]
\[ N = N_a + N_b \]
\[ N_a = N_b \]

\[ \langle W^2 \rangle = \langle (W_a + W_b)^2 \rangle \]

\[ \rho_{SF} = \frac{\langle W^2 \rangle}{4t\beta} \]

\[ S_{a,b}(k) = \sum_{p,q} e^{ik(p-q)} \langle n_p^{a,b} n_q^{a,b} \rangle \]
Ferromagnetic Phase Separation

OBC

\( \rho_a = 0.7 \)

\( \langle n_i^a \rangle = 1 \)

\( \langle n_i^a \rangle = 0 \)

\(
\begin{align*}
\langle n_i^a \rangle &= 0 \\
\langle n_i^b \rangle &= 1
\end{align*}
\)

\( \langle n_i^a \rangle - \langle n_i^b \rangle \approx 0.4 \\
\langle n_i^b \rangle \approx 0.6
\)

PBC

\( \rho_b = 0.7 \)

\( \langle n_i^a \rangle = 1 \)

\( \langle n_i^b \rangle = 1 \)

\( \langle n_i^a \rangle - \langle n_i^b \rangle \approx -1 \)

\( \langle n_i^b \rangle \approx 0.6 \)
Finite Temperature Results

\[ \beta = 1.0, \beta = 1.5, \beta = 6.5, \beta = 12.0 \]

\[ \rho \]

\[ \frac{\langle W^2 \rangle}{\beta t} \]

\[ \mu/t \]

\[ 3, 4, 5, 6, 7 \]
Phase Diagram of the doped system

\[ S(\pi, \pi) = L^{-\frac{2\beta}{v}} f \left( (T - T_{c}^{AF})L^{\frac{1}{v}} \right) \]

\[ \langle W^2 \rangle = \frac{4}{\pi} \]

\[ T_c(L) - T_c(\infty) \propto \frac{1}{\ln^2(L)} \]

\[ S(\beta, n) = S(0, n) + \beta E(n) - \int_0^\beta E(\beta', n)d\beta' \]

\[ \delta = \frac{N_a + N_b}{2L^2} - \frac{1}{2} \]
Findings

- Model is interesting and it has Competing phases.

- Found five distinct phases!
  - **AF phase** at half filling and low temperature
  - **AF to SF phase separation** near half filling
  - **NL phase** at finite temperature
  - **SF phase** for a broad region of filling
  - **Ferro-magnetic PS phase** inside SF phase at low temperature

- **PS dome inside SF phase shows greater entropy than AF and similar to the SF phase.**
  Experimental realization is possible!

Ferromagnetism with population imbalance of the species

\[ H = -t_a \sum_{\langle ij \rangle} (a_i^+ a_j + h c) - t_b \sum_{\langle ij \rangle} (b_i^+ b_j + h c) + U_{ab} \sum_i n_{i,a} n_{i,b} \]

- The prominent experimental challenge of quantum magnetic phases is to reach such low temperatures and entropies needed to observe these phases.

- The relatively high global entropy of the phase-separated ferromagnetic phase suggests that this ferromagnet should be easier to access experimentally.

- Then we thought to explore the extent of the phase-separated ferromagnetic phase as a function of finite polarization.
Extent of ferromagnetic phase with finite polarization

\[ \begin{align*}
L &= 10 \\
t_a &= 0.08 \\
t_b &= 1.0 \\
U_{ab} &= 6 \\
\text{Beta} &= 60
\end{align*} \]
Phase diagram on the optimal superfluid line

$L=10$
$\rho=1.25$
$P=0.25$
$\beta t=80$

$\rho = 1.5, P = -0.5 \ (N_a = L^2, N_b = L^2/2)$

$\rho = 1, P = 1 \ (N_a = 0, N_b = L^2)$
Ferromagnetic susceptibility

\[ \chi \sim L^{\frac{2}{\nu}} \rho \left( \frac{1}{L^{\frac{1}{\nu}}} (T - T_c) \right) \]

correlation length \( \nu = 1 \)

\[ R = \frac{\chi(0, \varepsilon) + \chi(0, -\varepsilon) + \chi(\varepsilon, 0) + \chi(-\varepsilon, 0)}{\chi(\varepsilon, \varepsilon) + \chi(-\varepsilon, \varepsilon) + \chi(\varepsilon, -\varepsilon) + \chi(-\varepsilon, -\varepsilon)} \]

\[ \mathcal{R} = \frac{\frac{1}{\beta} \int_{0}^{\beta} \sum_{j} e^{i k \cdot r_j} (n_j^a(\tau) - n_j^b(\tau)) d\tau.} \]

\[ \chi(k) = \langle |A(k)|^2 \rangle - \langle |A(k)| \rangle^2 \]
Superfluid density of light species

\[ \frac{\langle W^2 \rangle}{\beta t} \]

\( T_L/t \)

\( 1/\ln^2(L) \)

\( \rho = 1.25 \)

\( P = 0.25 \)
Superfluid density of heavy species

\[ \rho = 1.14 \]
\[ P = 0 \]
**k-space momentum distribution**

\[ N(0,0) = \frac{1}{L^2} \sum_{k,l} e^{i k \cdot (r_k - r_l)} \langle a_k^\dagger a_l \rangle \]
High Global Entropy for Ferromagnetic phase

\[ S(\beta, n) = S(0, n) + \beta E(\beta, n) - \int_0^\beta E(\beta', n) d\beta' \]
Findings

- Population imbalance between species extends the region of ferromagnetism.

- We found optimal superfluid line of fully phase separated ferromagnet with average local densities as $n_a \sim 0$ and $n_b \sim 1$ in one region and $n_a \sim 1$ and $n_b \sim 0.5$ on the other.

- **Ferromagnetic phase shows high global entropy**
  Experimental realization is possible!

Ferromagnetism with a harmonic trap

- The realization of magnetic phases with the presence of confinement is of interest in experiments involving cold atoms.

- We want to look for the existence of the ferromagnetic phase shown at finite doping and polarization with the presence of a harmonic trapping potential.

- We look for the ferromagnetic phase in both hard-core and soft-core limits.

\[
H = -t_a \sum_{\langle ij \rangle} (a_i^+ a_j + hc) - t_b \sum_{\langle ij \rangle} (b_i^+ b_j + hc) + U_{ab} \sum_i n_{i,a} n_{i,b} + W \sum_i (i - L/2)^2 (n_{i,a} + n_{i,b}) \\
+ \frac{U_{aa}}{2} \sum_i n_{i,a} (n_{i,a} - 1) + \frac{U_{bb}}{2} \sum_i n_{i,b} (n_{i,b} - 1)
\]
Hard-core limit

$L=40, W=0.06, N_a=450, N_b=675, U_{ab}=6t$
Soft-core limit

L=20, W=0.2, N_a=100, N_b=150, U_{ab}=6, U_{aa}=2.5, U_{bb}=1.75
Findings

- Such trapping effects can lead to having the ferromagnetic phase in both hard-core and soft-core bosons.

- In hard-core limit, we observe the phase separation where presumably the heavy species is localized in the center of the trap, and surrounded by the light species.

- In soft-core limit, we observe that the light species gets trapped in the center of the lattice, and is surrounded by the heavy species.

- The realization of the model is an open question for experimentalists. In collaborating with an experimental group, we are trying to figure out the precise range of parameters the experiments can reach.

Thank you!