Hysteresis, Avalanches, and Slow Relaxation: Complex non-equilibrium spin dynamics in a Zeeman-limited superconductor

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Outline:

1. Superconductivity and magnetic fields
2. The Spin-Paramagnetic phase diagram
3. Tunneling and excess single particle states near the S-P transition
4. Avalanches and FFLO physics
5. Summary

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Orbital Response of a Superconductor to a Magnetic Field

Bulk Superconducting Systems

Normal State

Superconducting State
(Meissner Effect)

Disk with width $<< \xi$ in a parallel field

Normal State

Superconducting State
(no screening currents)
Thin Film Superconductivity in High Parallel Magnetic Fields

Assume magnetic field oriented parallel to superconducting film of thickness $d < \xi$, so that there can be no significant orbital response to the applied field.

Energy:

- $\Delta$
- $2\mu_B B_{||}$

**S-P Transition**: 
(Spin-Paramagnetic) \[ B_{c||} \approx \frac{\Delta}{\sqrt{2}\mu_B} \sim 1.8 \text{ T/K} \] 
($g_L = 2$)
Electron Tunneling and the DOS

Tunneling Conductance: \( G \sim N_1 N_2 \left( kT \ll eV, \Delta \right) \)

\[ G \sim \frac{V \Delta}{e} \]

BCS density of states spectrum
**Sample Geometry**

3 - 5 nm Be or Al film

Glass slide

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**Al:**

$T_c = 1.1 \text{ K (bulk)}$

$T_c = 2.7 \text{ K (quenched film)}$

barrier type oxide $\text{Al}_2\text{O}_3$

g-factor $\sim 1.8^*$; $E_z \sim 1.8\mu_\text{B}B_{\parallel}$
S-P Phase Diagram of Pristine Al Films

Thin Al film in parallel magnetic field

\[ H_{c||}(T) \]

- \( t << \xi \)
- \( T = 70 \text{ mK} \)
- \( T_c = 2.8 \text{ K} \)

Marginally thin Al film in parallel field

\[ H_{c||}(T) \]

- \( t \leq \xi \)
- \( T_{tri} \)

\[ R (\Omega/\text{sq}) \]

\[ T (\text{K}) \]

S: superconducting
N: normal state
SM: state memory
S-I-S Tunneling

(a) 2.7 nm Al film/CE
$2\Delta_o = 0.93$ mV

(b) 5 nm Be film/CE
$2\Delta_o = 0.188$ mV

T $\sim$ 100 mK
Tunneling Probe of the Zeeman Field

R \sim 2000 \text{ ohms}

Zeeman Splitting of BCS DoS @ H_\parallel = 4.5 \text{ T}

Normal State DoS @ H_\parallel = 5.7 \text{ T}

Coulomb Anomaly \sim \ln(V)

Pairing resonance

\text{e-e interaction effect}
**Microscopic Nature of the Zeeman-limited Transition**

**Pure Al:** $H_{\text{app}} = 4.75$ T

Tunneling density of states of a 2.5 nm thick Al film in a parallel magnetic at $T = 80$ mK.

The origin of these excess states has been a mystery for more than 30 years.

1. Misalignment of $\sim 1$ deg
2. Leaky tunnel junction
3. Sample irregularities
4. Instrumentation issues

Note excess states at $V = 0$!

Red Curve: homogeneous BCS DoS
Inducing Exchange Fields in Al Films

Glass substrate

Applied field: $H_{\text{app}}$

EuS
FM semiconductor
$T_c \sim 16$ K
$\mu_0 M \sim 1.5$ T

$H_{\text{internal}} = H_{\text{app}} + H_{\text{ex}}$

Tunnel Junction

Al film
$$\Delta V = \frac{E_z}{e}$$

$$E_z = 2\mu_B H_Z$$

$$H_{\text{ex}} \sim 4 \text{T}!$$

$$eV^* = \frac{1}{2} \left( E_z + \sqrt{E_z^2 - \Delta_o^2} \right)$$

Aleiner and Altshuler, PRL 79, 4242 (1997)

$H_{ex}(T)$ vs $H_{app}(T)$ for different temperatures. The data points are shown for $80 \text{ mK}$ (circles) and $400 \text{ mK}$ (triangles).
Even with a pure Zeeman interaction, the excess states remain.

Al/EuS: $H_{\text{app}} = 0.03$ T; $H_{\text{ex}} = 4.3$ T

Again, excess states at $V = 0$!
our films as $d$ decreases from 3 to 2 nm, $D$ decreases by an order of magnitude, but $G(0)$ hardly changes. Furthermore, recent tunneling measurements of Al-EuS bilayers have shown that a comparable $G(0)$ is produced by an interface-induced exchange field, which is a pure Zeeman field with no orbital depairing effects [24].

Disordered LO states and excess low-energy spectral weight. —Having ruled out all the above explanations, we now argue that the anomalous excess zero-bias conductance at intermediate fields is an intrinsic property of the condensate due to the development of an exotic DLO phase with an inhomogeneous pairing amplitude and magnetization.

Our model consists of the attractive Hubbard Hamiltonian with a disorder potential and a Zeeman field:

$$ H = \sum_{rr'} t_{rr'} c_{r\sigma}^\dagger c_{r'\sigma} + \sum_{r} \left( V_r/\sqrt{N} \right) \left( n_{r\uparrow} - n_{r\downarrow} \right) \delta \left( n_{r\uparrow} + n_{r\downarrow} - 1 \right) + U/\sqrt{N} \sum_{r} \left( \left| c_{r\uparrow} \right| \left| c_{r\downarrow} \right| \right)^2; $$

(1)

where $t_{rr'}$ are hopping amplitudes (equal to $t$, taken as the unit of energy) between nearest-neighbor sites $r$ and $r'$, $n_{r\sigma} = c_{r\sigma}^\dagger c_{r\sigma}$ is the number operator for fermions of spin index $\sigma = \uparrow/\downarrow$ at site $r$, $\delta(n_{r\uparrow} + n_{r\downarrow} - 1)$ is the average chemical potential, $h$ is the Zeeman field, and $U$ is the local pairwise Hubbard interaction. The disorder potential $V_r$ at each site is picked independently from a uniform distribution on $[-W/2,W/2]/C0$. We calculate the local densities $n_{r\sigma}$, pairing amplitude $\Delta = |U|h c_r^\dagger c_r$, and spin-dependent DOS $N(\sigma)(E)$.

FIG. 3 (color online). The first two columns show spatial maps of the local pairing amplitude $\Delta$ and the magnetization $m$. The third column show the DOSs of up and down electrons $N(\sigma)(E)$. The last column shows the total DOS $N(E)$. For intermediate fields (e.g., $h/t = 0.95$ and $h/t = 1.2$) the system exhibits disordered Larkin-Ovchinnikov states with domain walls at which $m$ is finite, $\Delta$ changes sign, and the DOS becomes finite at low energy. Other parameters are as in Fig. 2.

FIG. 4 (color online). (a) Combined plot of $m(r)$ and $\Delta(r)$ for $h/t = 1$ (other parameters as in Fig. 2). Red (blue) indicates regions where $\Delta(r)$ is large and positive (negative). Brown regions, where the magnetization $m(r)$ is large, occur at domain walls where $\Delta$ changes sign. White regions are hills or valleys of the disorder potential corresponding to empty sites or localized pairs that participate in neither superconductivity nor magnetism. (b) and (c) show oscillations of $\Delta$ along the vertical dashed line in (a). (d) and (e) show the correspondence between magnetization $m(r)$ and low-energy spectral weight $I(\tau) = 1/2\pi \int dE N(\sigma)(E)$.

Avalanches and Slow Relaxation Near $H_{c2}$

JCP098B: Al-TJ (568 $\Omega$/sq), $T=53$ mK, $\theta = -0.5^\circ$
Two Identical Runs

R/$R_n$ vs. Angle - JCP098B
($R_s = 568 \Omega$/sq, $T = 52$ mK)

- 3.0°
- 1.5°
- -0.5°
Avalanches in the DOS

Al film: \( t = 2.5 \text{ nm}, R = 540 \text{ Ohm/sq} \)
Statistics of the Avalanches

\[ T_{\text{Tri}} = 750 \text{ mK} \]

\[ T_H = \frac{2\mu_B(H - H_0)}{k_B} \]
Minor Hysteresis Loops

Figure 6. The $53\text{mK}$ minor hysteresis loops of the zero-bias tunneling conductance as a function of the parallel field for an $542\Omega/\text{sq}$ Al film, normalized by the normal-state value ($G_{n}$). The major loop is shown in black. Arrows indicate the field sweep direction. Upper panel: The field sweep was initiated from a hypercritical field and then swept back and forth between the same two parallel fields ($6.02 \leq H_{\parallel} \leq 6.2$). Lower panel: The field sweep was initiated from a subcritical field. With each subsequent loop, the field is swept closer to the upper critical field and then returned to the initial subcritical field.
our films as dis decreased from 3 to 2 nm, D decreases by an order of magnitude, but G\(_0\) hardly changes. Furthermore, recent tunneling measurements of Al-EuS bilayers have shown that a comparable G\(_0\) is produced by an interface-induced exchange field, which is a pure Zeeman field with no orbital depairing effects [24].

Disordered LO states and excess low-energy spectral weight. —Having ruled out all the above explanations, we now argue that the anomalous excess zero-bias conductance at intermediate fields is an intrinsic property of the condensate due to the development of an exotic DLO phase with an inhomogeneous pairing amplitude and magnetization.

Our model consists of the attractive Hubbard Hamiltonian with a disorder potential and a Zeeman field:

\[
H = \sum_{rr'} t_{rr'} \hat{c}_{r}^\dagger \hat{c}_{r'} + \sum_{r} \left( \alpha + \frac{h}{c} \right) n_r - \sum_{r} V_r n_r - \sum_{r,r'} \frac{U}{2} \hat{c}_{r}^\dagger \hat{c}_{r}^\dagger \hat{c}_{r'} \hat{c}_{r'},
\]

(1)

where \(t_{rr'}\) are hopping amplitudes (equal to \(t\), taken as the unit of energy) between nearest-neighbor sites \(r\) and \(r'\), \(n_r = \hat{c}_{r}^\dagger \hat{c}_{r}\) is the number operator for fermions of spin index \(s\) = 1 at site \(r\), \(\alpha\) is the average chemical potential, \(h\) is the Zeeman field, and \(U\) is the local pairwise Hubbard interaction. The disorder potential \(V_r\) at each site is picked independently from a uniform distribution on \([0,W/2]\). We calculate the local densities \(n_r\), pairing amplitude \(\Delta_r = \sqrt{U} c_r^\dagger c_{r'}\), and spin-dependent DOS \(N_{\uparrow}(E)\) and \(N_{\downarrow}(E)\). The last column shows the total DOS \(N(E)\).

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Summary

- We believe that the SC order parameter is non-trivial in the hysteretic region of the S-P transition.
- Excess states at the Fermi energy may be an indication of a disordered FFLO phase that emerges in a high Zeeman field.
- Tunneling data shows that the avalanche behavior is in the condensate itself.
- The asymmetry of the avalanches is unusual and cannot easily be explained.